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An exact theory of the industrial evolution and national recovery

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Preface

We started this work in a case study for graduate students, convinced that an exact theory of economic growth with forecasting quality must and can be found. Subsequently most of our decisions in industrial research and development had to be made without academic backing. Some of them led us to question fundamental assumptions. For instance, we learned that technical progress cannot be responsible for the pace of macroeconomic growth. The level of education and the last century's reservoir of relevant but unused patents and prototypes were convincing and growing, although the real long term national growth rates per capita of leading nations decreased since 1955.

The investment question of how long China would continue to grow with real 10 percent p.a. per capita got us back into the fundamental problems of economic growth theory. We solved some of them with the discovery of a causal connection between the depth of a disaster and the stability of recovery. The necessary and sufficient conditions for success were complete renormalization of the economy, social coherence, and evolution in peace.

Our theory and first predictions were published 12 years ago although we did not understand why just 3 decisive parameters determined long term growth, and what kept them constant from 1800 to date. Their continued constancy is decisive for the quality of predictions. Back in academia this work benefited from the discovery of an S-functional parallelism between the mean life expectancy and the envelope of the best real GDPs per capita above all political and financial disasters. This identified the decisive parameters as genetically stabilized constants of the human species.

Therefore the present version of our natural theory has full forecasting power unless global resources supersede human biology in limiting the pace of economic growth per capita. In retrospect this was to be expected for a species at the end of the food chain guided by natural science.

Contents

1. Introduction: Basic and absolute problems of macroeconomic forecasting

The financial crisis of 2008 and the return to Maynard Keynes' policy for recovery from the Great Depression showed that the quantitative understanding of economy and technology is not really converging. This comparison is appropriate because there is agreement, from Adam Smith's blueprint of the industrial society [1] to Robert Lucas' [2] and Paul Romer's [3] inclusion of educational and developmental inputs, that economic growth is driven by technical progress, which means by knowledge obtained with natural science. Every student feels the strange challenge of this qualitative gap because nothing can successfully work against the laws of nature, not the economy, and definitely not the financial world.

Since the success of natural science and technology is based on the forecasting quality of exact solutions and natural constants, the method for closing the gap in understanding is clear. The observation of linear economic convergence in Europe [4] gave the incentive for deriving our first exact solutions without adjustable parameters [5, 6]. They include the recovery of the USA from the Great Depression, of Europe and Japan from WW II, of Korea from the Korean War, and the collective industrial evolution above all disasters from 18th century UK to date. Their comparison with the real GDP data per capita is shown in the lower part of Figure 1 of Section 3.

That these advances could be obtained only recently, and beyond mainstream theory, has two different reasons. Their novelty is due to the relatively new data shown with Figure 1. They were not available in 1956 when Robert Solow presented his seminal differential equation for generating physical capital by investing a constant fraction of the GDP [7]. The exact theory generalizes this equation with a fundamental term which was difficult to see for Solow's focus on the USA, but decisive for the fast recovery of Germany and Japan from WW II.

The reason for the need to go far beyond mainstream economic theory is historical. Natural science started with macrophysics and the search for the underlying laws of nature, using Leibniz' and Newton's analysis for quantitative understanding. Natural science developed hereby a theoretical culture marked from the beginning by the experience that a system can be so independent of its substructure that it has its own laws and parameters, and that its behaviour is exactly predictable when these parameters are constants of the system.

Economic theory started with microeconomics and remained in its complexity which all but excludes exact solutions. Keynes was the first to comment on our habit of converting exact mathematics into approximations by designing economic properties into them with adjustable parameters. But forecasting quality needs exact solutions including economic properties in their very structure without any adjustable parameter. So far the advantages of such solutions could not be appreciated in economics, but the above examples show that they exist for macroeconomic production.

Yet the absolute problems of economic forecasting do not appear on the production side. They are due to the fact that it is impossible to measure the benefits of the goods and services for family life at home. Since these benefits compete for the overall result, they are not additive like money, so that we don't even have compatible scales, coordinates, and variables for measuring demand in terms of a macroeconomic user value of the GDP. This explains why we could so far neither set up an exact condition for dynamic equilibrium between demand

and supply nor find any continuously valid time dependence for life at home or the amount of labour required for maintaining economic equilibrium.

Adam Smith saw these absolute problems clearly in his vision of a free market because instead of finding excuses he called upon his famous “invisible hand” for achieving economic equilibrium between demand and supply. After 250 years we must admit that it is simply impossible to derive an exact growth theory within mainstream economics.

The following sections show that such a theory, including life at home, is possible on the highest aggregate level with basic information from biology, physics, and industrial research and development. We can use of course the data listed in the national statistical reports for the production variables GDP, working time, and physical capital, because they are exact for two reasons: Costs and profits can be measured with money so that their values can be easily added up for any purpose; and the constancy of the exact theory’s parameters over 8 generations proves that all production values were astonishingly well corrected for inflation.

Main results: Forecasting quality requires an exact theory with constant parameters. This triad can only be obtained beyond mainstream economic theory. The goal is finding a self-consistent and independent frame for macroeconomic theory without severing its microeconomic connections.

2. Obtaining exact solutions with natural constants for economic growth

Since the following sections focus on economic aspects and results, this section introduces into some general concepts of flow systems with storage capacities. Their main advantages are that they unite the information from four academic disciplines and practical experience, determine the structure of the theory, and specify the data required for verification.

Maintaining life requires a steady flow of goods and services per unit of time. The flow begins with global resources and ends as waste, including its partial reuse and temporary storage in individuals and physical capital. Since we will find out that the industrial evolution was so far limited by human biology and not by technology, environment, or the universal laws of nature, we postpone the early Club of Rome’s environmental feedback scenarios until they become so serious for the then leading nations that reliable data and feedback paths are available and respected for survival.

Symmetry and conservation are the most valuable general concepts. Using them is the only way for resolving the absolute problem of macroeconomics, namely the lack of data, scales, and coordinates for life at home. Symmetry contributes structural information. Since Adam Smith’s concern with industry’s separation between home life and factory work it was clear that a complete theory must have two sides with a fundamental symmetry for generating and using the flow of goods and services. Since the value of physical capital $k(t)$ per capita stores the technical knowledge required for production, there must exist a counterpart storing at least the knowledge required for selecting and using the goods and services for life at home. We name it “human capacity” $h(t)$ per capita in order to prevent confusion with the “human capital” used some time ago for the production value of the work force.

The new quantity *h(t)* is central to the exact theory. Section 5 shows that its formal value follows from the investment in education, and that it stores the industrial society's current level of relevant knowledge. Its technical subset is stored in physical capital *k(t)*, but the latter can be duplicated just with money, whereas increasing *h(t)* requires new knowledge which takes more time because it requires social interaction. Therefore we have generally $k \geq h$.

Fortunately two conservation laws exist for flow systems. The first is that the GDP *y(t)* per capita, the annual flow of goods and services, is the same for using and producing. This identity saves one variable. The second is that we have not one year for producing and another for using the GDP but the same time is flowing by for both. Section 9 shows that unpaid working time at home, including reproduction, balances paid working time for production so that there is a trade-off between working time *w(t)* and spare time *s(t)* per capita and year.

That this trade-off was all but neglected so far is surprising because even to-day's affluence would be useless without the spare time for using and enjoying it. With 8 hours of sleep and 6 working days per week the maximum weekly working time is 96 hours. Appendix A1 shows that this was in fact the agricultural and early industrial working time around 1800. Taking the maximum of 96 hours per week as the unit $\bar{\varepsilon} = 1$ p. a. for measuring annual working time, we have the fundamental relation

(1) $$s = \bar{\varepsilon} - w$$

for annual spare time at home. Figure 5 in Section 9 shows that working or spare time, life expectancy, and the evolutionary GDP are cultural variables of the industrial society and, as such, nearly immune even to world wars.

The number of relevant variables is herewith reduced from six to four, with *h(t)* only one more than the known production variables *k*, *w*, and *y*. We have learned also that the "reproduction factors" human capacity and spare time are as important for using the GDP as the production factors physical capital and working time are for producing the GDP.

The last structural information we get from general concepts of flow systems is the number of possible dynamic responses. For one-dimensional variables it is limited to one response per independent variable. Therefore we can have with *k* and *h* only two identifiable dynamics. Since *k* is destructible it must be responsible for recovery from disaster in Figure 1. *h* must then be responsible for the industrial evolution. The latter's immunity even to world wars shows that human capacity must be per capita indestructible. This means that h is in contrast to *k* an inseparable part of every individual.

When *k* and *h* are kept constant, *w* and *y* can still support an oscillation between them which can appear as business cycles. Section 12 shows why they are so rare, but we obtain the economy's reaction time to external shocks from the exact and complete solution for cycles.

Since the pace of every dynamic response must depend on the storage or reaction time of the quality behind the responsible variable, these times must explicitly appear in every differential equation for exact solutions. Sections 5 and 6 identify these times for *h* and *k*. For *w* and *y* it is by definition the calendar year $1/\varepsilon$.

<u>Main results:</u> Applying the general concepts of symmetry and conservation to flow systems can resolve the absolute economic forecasting problem. They also determine the structure of the theory, the number of variables, and the kind of data required for verification.

3. The industrial evolution, relevant knowledge, and national recovery

A priori we can get exact solutions only for per capita growth. The simple reason is that the birth rate is not exactly predictable. Per capita growth can however be readily scaled up to national growth after the birth rate is known because the life expectancies are very well known. The population of affluent nations tends to stabilize when it is not already declining as in many European nations and Japan.

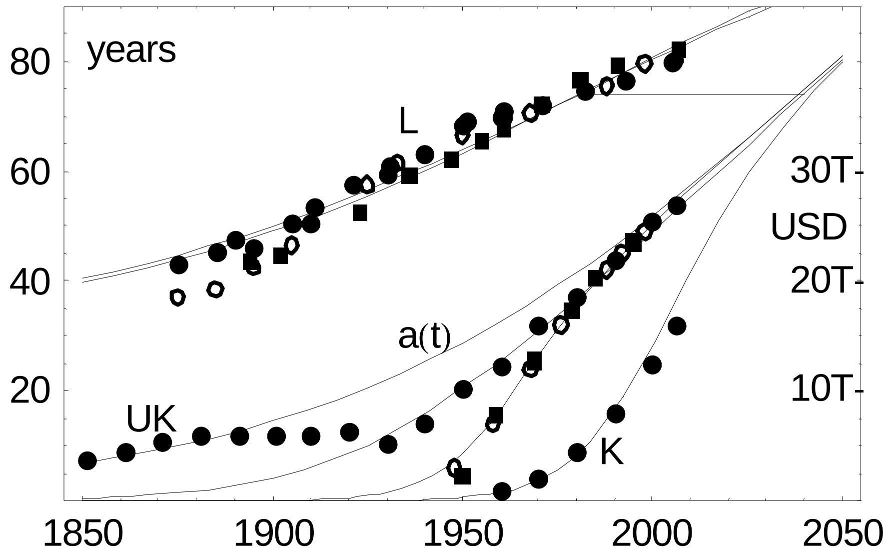

Figure 1: Unisex life expectancy L (left hand scale), recovery of the real GDPs per capita of leading nations (right hand scale in the US $ value of 1991), and their convergence into the industrial evolution a(t) compared with theory (plots). Data points are for the UK and the USA, circles for Germany, and squares for Japan.

Figure 1 shows in the lower part the national recoveries *y(t)* of the real GDPs per capita and their convergence into the collective industrial evolution *a(t)* established in 18th century UK [5]. Growing competition from Continental Europe and the USA caused the UK's stagnation between 1890 and the Great Depression of 1929. The latter caused the first complete recovery of the USA. The second is the recovery of Germany and Japan from WW II. South Korea's recovery from the Korean War is presently at half time of convergence.

The GDP connects production with life at home because the flow of goods and services per capita determines directly the quality of life. Its improvement is the driver for research and investment in human capacity and physical capital. Therefore we start deriving the exact theory with our species' talent for copying relevant knowledge from nature for producing the GDP of the industrial evolution. All data in Figure 1 exclude exponential growth with constant rate $\dot{a}/a$ as valid long term growth law. Fortunately there is another differential equation expressing this talent without requiring an extra parameter:

(2a) $$\dot{a}/a = (1 - a/\bar{a})/E \, .$$

Its general solution is the S-function

(2b) $$a = \bar{a}\Big/\left(1 + e^{(T_a - t)/E}\right)$$

plotted in Figure 1 with half time in $T_a = 2040$. The "evolution constant" $E = 62$ years is the inverse of the initial growth rate for $a \to 0$ and $t \to -\infty$. Growth is nearly exponential until $t \le T_a - 2E$, and within $T_a \pm E$ nearly linear with $\dot{a} \cong \bar{a}/4E$. The asymptotic final GDP for $t \to +\infty$ is $\bar{a} =$ 75,000 US $ p.a. per capita in its value of 1991. For exponential growth with any positive constant rate the relative gap $(\bar{a} - a)/\bar{a}$ to be closed would be constant (equal to one since $\bar{a} = \infty$), and the ability a/E for increasing $\dot{a}$ would go to infinity which is per capita impossible.

(2a) describes the result of a complex process. Since leading nations have no superior nation to copy from, they must obtain new knowledge from nature with research. But knowledge has no economic value by itself because it does not wear out by being used. It must acquire such value with embodiment in *h(t)* by education and in *k(t)* by design. This means successful competition with established knowledge. Within the evolution constant *E* both can become socially coherent. The resulting current mix constitutes the relevant knowledge, the conscious addition superimposed on the inherited capacities. Daily experience shows that they are intrinsically connected and include the interests of three consecutive generations.

This entire process, including its effective control by selecting and using the goods with human capacity *h(t)* at home, will be quantified in the following sections. It is irrelevant which nations and how long they are lucky or good enough to stay at the evolutionary level. Essential is only a sufficiently large reservoir of relevant technical knowledge so that (2a) sets the pace. Too much or too advanced physical capital is just idling, not enough or outdated physical capital results in a general GDP of $y(t) < a(t)$.

Recovery from disaster or catching up with leading nations is actually easier and faster than staying at the evolutionary front, because the relevant knowledge stored in human capacity per capita is indestructible, and new knowledge can be copied from leading nations. The corresponding differential equation for recovery and convergence into (2b) is therefore given by

(3a) $$\dot{y}/y = \beta(1 - y/a) + (\dot{a}/a)y/a\,.$$

Its general solution $y^{-1} = a^{-1} + \bar{a}^{-1}e^{\beta(\tau_y - t)}$ is surprisingly simple because in reciprocal economic space the exponential functions appear in the nominator. This is a general property of the exact theory in analogy to the fact that the inherited genetic program reels off during life, so that the removal of obstacles and human incapacities for realizing the full program appears now as driver of growth. In this space the fundamental difference to the exponential function is that the latter does not approach a smallest but a zero level of incapacity. This is excluded because it would mean omnipotence of the human species.

Inserting (2b) into the general solution yields the double S-function

(3b) $$y = \bar{a}/(1 + e^{(T_a - t)/E} + e^{\beta(\tau_y - t)})$$

for successful recovery and convergence into the industrial evolution. The convergence parameter β is according to (3a) the initial growth rate for $y << a$. Section 6 shows that it is quantitatively determined by the national renormalization condition for recovery.

(3b) is plotted in Figure 1 with the following values for β and halftimes of recovery:

USA 0.05 p.a. and 1965, Germany and Japan 0.09 p.a. and 1971, and Korea 0.08 p. a. and 2005. The nearly linear growth phase before convergence was already observed by Crafts and Toniolo in 1996 for European nations [4]. It is due to the long linear phase of the evolution and its extension by the recoveries seen in Figure 1.

Main results: The exact theory replaces the steady state solution of exponential growth with the exactly known S-function of the industrial evolution per capita. The latter is immune even to world wars and approaches asymptotically a steady state with zero growth rate per capita. Every successful recovery from disaster converges with the industrial evolution.

4. The life expectancy and the biologic stabilization of economic growth

Several completed recoveries show that the exact solution (3b) is valid and relevant, but there can be only one industrial evolution, so that the constancy of E and $\bar{a}$ for over 8 generations was mysterious. That this constancy can hardly be the result of human control led in due time to our discovery of the parallelism between *a(t)* and the mean life expectancy shown also in Figure 1. Finally the hereditability of longevity [8] supported our conclusion that E and $\bar{a}$ are also biologically stabilized [9].

The data in Figure 1 support at first sight the life insurers' linear extrapolation of national life expectancies. But this can only mean that the life expectancies are presently in the linear phase of a biologically acceptable function. Fitting therefore an S-function to the data yields

(4) $$L = L_o + (\bar{L} - L_o)\big/(1+e^{(T_L - t)/E}) \equiv L_o/(1+e^{(t-T_L)/E}) + \bar{L}/(1+e^{(T_L - t)/E}).$$

It is plotted in Figure 1 with the same evolution constant of E = 62 years, halftime in $T_L = 1981$, minimum life expectancy for successful reproduction $L_o = 30$ years, and an asymptotic final life expectancy of $\bar{L} = 118$ years. The latter specifies for the leading nations the generally expected mean final life expectancy of about 120 years. The slightly flatter plot for *L(t)* is the theoretical result derived immediately.

The S-functional identity (4) means that dropping obsolete and accepting new elements of relevant knowledge is a social process requiring the same reaction time *E*. This is compatible with our assumption that decisions in life are based on the current mix of new and established knowledge. It also means that knowledge is not an additive quality, neither for individuals nor for society. Since conflicting interests coexist in individuals as well as between generations, the main goal of education is maximizing the overlap of individual relevant knowledge.

The life expectancy is the lasting result of selecting and using the goods and services with the current set of relevant knowledge embedded in *h(t).* It is our first and so far only quantitative information for resolving the absolute economic problem of quantifying the user side of the economy. We start with understanding the parallelism between *L(t)* and *a(t).*

The obvious and simplest possible assumption is that life integrates and averages over the existential conditions given by the evolutionary GDP. This is legitimate because life insurers correct the mortality tables for catastrophic deaths, and the evolutionary GDP is the envelope above all disasters. The exact integral exists with the averaged result

(5) $$L = L_o + \frac{\overline{L} - L_o}{\overline{L}\,\overline{a}} \int_t^{t+\overline{L}} a\,dt = L_o + \frac{E(\overline{L} - L_o)}{\overline{L}} Log \frac{1 + e^{(t+\overline{L}-T_a)/E}}{1 + e^{(t-T_a)/E}} .$$

It is the second plot in Figure 1 with the slightly smaller slope at halftime T_L. In fact, (5) has the very good S-functional approximation

(6) $$L = L_o + (L - L_o) \big/ (1 + e^{(T_a - \overline{L}/2 - t)/E}) .$$

Comparison with (4) yields for their time shift $T_a - T_L = \overline{L}/2 = 59$ years. This means that we must not wait 200 years or extrapolate *L(t)* for the maximum life expectancy $\overline{L}$, we have measured it already with the observed time shift $\overline{L}/2$. The halftime bar for both S-functions in Figure 1 shows these 59 years.

That the industrial society evolved already past the inflection point at halftime, where its annual increase had its maximum of $\dot{L} = (\overline{L} - L_o)/4E = 0.35$ years per calendar year, explains again why an exact theory of economic growth could not have been obtained earlier.

Comparison between (4) and (6) yields the main result of this section: *L(a)* is the simple affinity transformation

(7) $$L = L_o + ((\overline{L} - L_o)/\overline{a})\, a\,(t \to t + \overline{L}/2)$$

named "bioeconomic relation". It is absolutely stable because it contains with L_o and $\overline{L}$ only two parameters which are constants of our species.

(7) means that human biology stabilized and limited economic growth at any time and in every culture from the agricultural affluence of Mesopotamia and Egypt to the present industrial evolution. The constancy of the "bio-economic ratio" $(\overline{L} - L_o)/\overline{a} = 1.17$ years per 1.000 US$ in the value of 1991 means that everyone has in the ensemble average an asymptotic limit $\overline{a}$ for sensibly digesting his and her mix of goods and services with personal gain. The best part of accumulated surplus is actively used by employing other people or given to charity. A large part was so far subject to decay, destruction, or inflation. The rest was non-sensibly used or even punishable.

Main results: The evolution of the industrial society is biologically stabilized and immune even to world wars. The presently responsible generation lives between the halftimes where the evolutionary GDP and the life expectancy have the largest annual increase. The stability of the bio-economic relation shows that human behaviour follows nature in not separating between academic disciplines.

5. Investment in education, human capacity, and the invisible hand

The life expectancy follows quantitatively from (5), life's integration and averaging over future existential conditions. On the other hand, the time dependence of the evolutionary GDP is given by the life expectancy's biologically stabilized evolution constant. This raises the question of causality, especially since an averaging process cannot be reversed without prior knowledge of the original function. The obvious solution is the existence of a third so far hidden quantity guiding *L(t)* and *a(t)* nearly simultaneously. This means that Adam Smith's invisible hand really exists. Since Section 2 showed that the exact theory requires only one more quantity, we have no choice: The hidden quantity must be identical with human capacity *h(t)* which embodies Adam Smith's invisible hand. What "nearly" means will be specified in Section 10 with the self-consistent time shifts between *h(t)*, $\overline{L}(t)$, and *a(t)*.

Since leading a successful life requires more than just technical knowledge, *h(t)* must store the industrial society's entire relevant knowledge. Only its technical subset is stored in *k(t)*. The annual maintenance costs of both storable quantities must be part of the GDP. Figure 2 shows these parts for three leading nations. The costs of public order are also shown because all three are indispensable operational costs of the industrial society. The plots within the public order data represent the cost shares derived in the next section for the maintenance of physical capital. The data for public order and education were compiled from the national statistical reports except for Japan's educational costs [10].

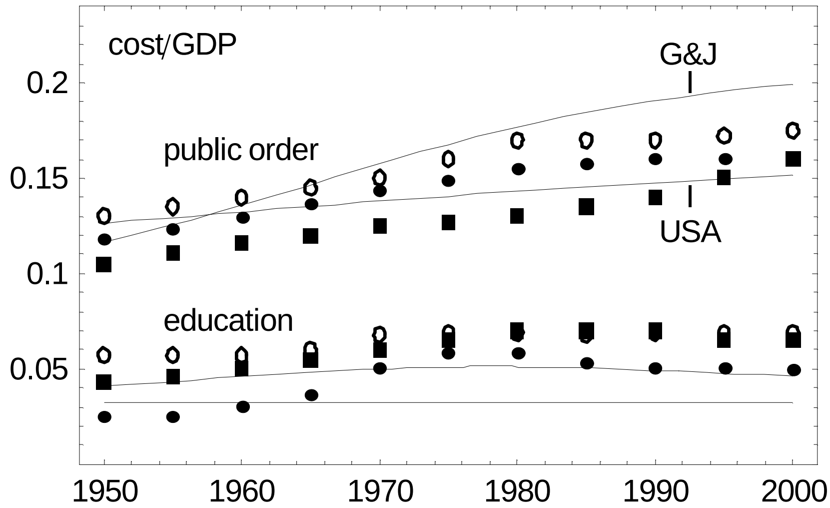

Figure 2: Cost of education (data and plots), public order (data), and maintaining physical capital (upper plots) per GDP for Germany (points), Japan (squares), and the USA (circles)

The public order costs include every directly related effort from legislature and the judiciary to the national and private security systems. That they increase in parallel with the plots for the maintenance of physical capital is understandable because the unequal distribution of wealth was and will remain the biggest challenge for national order and international peace. That Japan has the lowest ratio of public order to physical capital costs and the USA the highest is mainly due to the large difference in their military effort. The educational costs include public and private costs from kindergarten to university.

All three fixed costs amount already to 40 % of the GDP. Since the next section's investment for increasing physical capital must be added, only about half of the GDP remains for private consumption including housing, health and provisions for retirement.

The actual values of *h* and *k* can be derived from the annual cost of maintaining them. This defines for human capacity with

(8) $$h/E = \nu\, y$$

the “capacity share” $\nu(t)$ of the GDP, corresponding to the lower data in Figure 2. Their wide spread is due to the fundamental difficulty of measuring the efficiency of education and the individual value of *h*. But this is uncritical because the parts of human capacity and physical capital actually needed at home and at work are exactly determined in Section 10 from the GDP and annual working and spare time.

The relevant time constant for maintaining the level of *h* is the evolution constant *E* because what matters more than any amount of knowledge is its social coherence discussed already in Sections 3 and 4 for (2a, b) and (4).

The maintenance balance

(9) $$k/G = \mu\, y$$

for physical capital defines the “capital share” $\mu(t)$ of the GDP shown with the upper plots in Figure 2. It is derived in the next section. The European Central Bank uses $G = 25$ years for the physical lifetime of *k* against wear and obsolescence [11]. This is close enough to the reproduction cycle that we named it “generation constant” [5]. *G* completes with *E* and $\bar{a}$ or $\bar{L}$ the biologically stabilized set of socioeconomic constants. Every generation takes essentially care of its own technical infrastructure. An inglorious exception is former East Germany. It had to be wound up in the end because new physical capital was financed under governmental order with increasingly deficient maintenance of the existing physical capital.

y(t) is defined in the full range $0 \le y \le \bar{a}$ by (3b). The corresponding range $0 \le k \le \bar{\mu}\, G\, \bar{a}$ of (9) defines with the next section’s total capital share $\bar{\mu}$ the asymptotically approached final state $(\bar{a}, \bar{k})$ of the industrial evolution. Its initial state is not so simple. Industrialisation started from the agricultural and guild society where the family was the economic unit. *h* and *k* were not yet separated, life was at subsistence level, and annual working time at its maximum $\bar{\varepsilon} = 1$ p. a. according to (1). Appendix A1 describes the cultural transition of separating work from home and *k* from *h* as a programming exercise.

Recovery starts from the level of physical capital left after destruction, but *k* and *h* are already separated, and Section 2 explained why $k > h$. This means from (9) that for $\mu > 1/\varepsilon G$, i. e. for net growth, there must always be a point in time of total symmetry where $h_o = k_o$, $y = y_o$, and $s_o = w_o = \bar{\varepsilon}/2$ from (1). This happens at relatively low economic levels. We can use this point for calibrating the capacity and capital shares. *k* and *h* are designed and educated as amplifiers of working time for producing the GDP and of spare time for using and enjoying it, respectively. We must take the entire GDP because the contributions of public order, education, and physical capital shown in Figure 2 are also improving the quality of life. Making the simplest assumption of linear amplification because it does not require any unknown adjustable parameter, we obtain

(10) $$\bar{\varepsilon}\, h_o/2 = y_o = \bar{\varepsilon}\, k_o/2$$

for economic equilibrium. Comparing (10) with (8) and (9) yields for the initial capacity share

(11a) $$\nu_o = 2/\varepsilon E = 0.032\,,$$

and for the initial capital share

(11b) $$\mu_o = 2/\varepsilon G = 0.080\,.$$

(11a) is shown with the straight line in Figure 2. The plot above this line is a better fit to the educational data. But the difference to the straight line was essentially the Western answer to the "Sputnik shock", following the first satellite orbited in 1958 by the former Soviet Union. It consisted mainly of extending university laboratories and research staff. This did not increase human capacity greatly for the user side, and whatever it may contribute takes still the evolution constant for relevance. Therefore we accept (11a) for (8) as theoretical level for *h(t)*. Considering that there was so far no quantitative clue at all to a rational level of educational investment we may be satisfied with this kind of agreement.

<u>Main result:</u> Human capacity embodies Adam Smith's invisible hand. The cost of public order correlates with the uneven distribution of wealth. The nation's educational effort in Figure 2 appears to be steady and reasonable except for Germany.

6. Investment in physical capital and its trade-off with the growth rate

The time derivative of (9) is $\dot{k} = \mu G \dot{y} + \dot{\mu} G y$. The total amount of support required for annual addition and maintenance of physical capital is therefore

(12) $$\dot{k} + k/G = \mu\,(1 + G\dot{y}/y)\,y + \dot{\mu} G y = \overline{\mu}\, y + \dot{\mu} G y\,.$$

This is the generalized form of Solow's seminal differential equation for the generation of physical capital from the GDP. The "total capital share" $\overline{\mu}$ corresponds to Solow's constant saving ratio. It includes the maintenance term k/G and the term $\mu G \dot{y}$ which keeps the capital coefficient k/y constant. The new "capitalization term" $\dot{\mu} G y$ is responsible for increasing the capital coefficient. It was neglected to date because it was too small to be observed for the USA. But it contributed up to 40 % to $\dot{k}$ for the fast recovery of Germany and Japan from the nearly total loss of physical capital in WW II. It was observed for the first time in the investment as well as in the structural inflation [5, 6] explained in Section 13.

With $\mu = \mu_o$ from (11b) and the long term average of $\overline{\mu} = 0.18$ for the USA one obtains from (12) for Solow's steady state growth rate $\sigma \equiv \dot{y}/y = (\overline{\mu}/\mu_o - 1)/G =$ 0.05 p.a. for *y* and *k*. This agrees with the mean growth rate of the USA for the initial phase of recovery from the Great Depression plotted in Figure 1.

The existence of the capitalization term allows $\overline{\mu}$ to be exactly constant and the balance (12) exactly valid for the full range $-\infty < t < \infty$. This specifies a permanent trade-off between increasing *k* and decreasing growth rate $\dot{y}/y$. Since this is also the intrinsic property of the differential equations (2a) and (3a), the exponential function must be replaced by the S-function as the natural, exact, and general solution for economic growth.

Every economy can grow in principle until its total capital share is used up for maintaining the accumulated physical capital so that nothing is left for further growth. That poor nations grow faster than rich nations is no longer a mystery but the law for equally diligent nations. (12) defines also the optimal path of recovery because it maintains the value of accumulated physical capital. $\overline{\mu}$ is stabilized with depreciation and investment. The former is the most effective incentive for maintaining the value of physical capital, the latter is stabilized because banks defend their share of the economy by advocating reinvestment.

Diligent nations fix $\overline{\mu}$ at the maximum possible level. The record has China with $\overline{\mu} = 0.28$. Germany. Japan, and Korea recovered with 0.25 ± 0.01. The USA converged with only 0.18 because the Great Depression was not a loss of real physical capital, and because the USA are still consumption oriented with a relatively large agricultural sector.

Solving (12) for the capital share yields

(13) $$\mu = \overline{\mu}/(1 + G\dot{y}/y)\,.$$

It is shown with the plots for the USA and Germany & Japan in Figure 2 with the data listed below (2b) and (3b). Inserting the initial state ($\dot{y}/y = \beta$, $\mu = \mu_o$) into (13) yields with (11b)

(14a) $$\overline{\mu} = \mu_o(1 + \beta G) = 2\,(\beta + 1/G)/\varepsilon$$

and its reversal

(14b) $$\beta = \overline{\mu}\varepsilon/2 - 1/G\,.$$

k(t) is now exactly determined by (3a, b), (9), (11b), and (13). Besides the historic positions of T_a and τ_y on the time scale only the four parameters *E*, *G*, $\overline{a}$, and $\overline{\mu}$ determine the time dependences of *y* and *k*. The first three are constants of our species so that all successful recoveries must converge into one and the same collective industrial evolution. The recoveries in Figure 1 look so similar because the only free parameter for a national difference is $\overline{\mu}$ in the range $0.18 \leq \overline{\mu} \leq 0.28$.

Main results: The capitalization term generalizes Solow's differential equation. S-functions are the natural and exact solutions for economic growth. The trade-off between growth rate and wealth is an inevitable consequence of the increasing cost of maintaining physical capital.

7. Self-consistency and constant time shifts between variables

When a storable quantity like physical capital $k(t)$ gains or loses value with a positive or negative flow $\dot{k}(t)$, the memory effect of the quantity's finite reaction or storage time causes different time dependencies for both variables, and well defined time shifts between them. The latter are an intrinsic property of the economy and usually also constants. There are three regular cases:

Forced or assumed exponential growth and decay produce the same shape for all time derivatives and integrals, but the exponential function's unlimited exchangeability between amplitude and time prevents also the detection of existing time shifts. Regular oscillations produce constant time shifts which are easily observed as phase shifts (Section 12). S-functional growth causes different shapes for every integral and differential, but all time shifts are constant. They can be easily measured when the variables are normalized to the same amplitude.

This means for the exact theory that the time shifts between GDP, human capacity, physical capital, and their integrals and differentials are exactly observable, and that they are completely determined by *E*, *G*, and β or $\overline{\mu}$. Agreement of the theoretical shifts with reality is therefore an automatic self-consistency test of our theory.

Figure 3 shows the time shift between y and k for the UK, USA, and West Germany as examples for evolutionary, smooth, and fast growth, respectively.

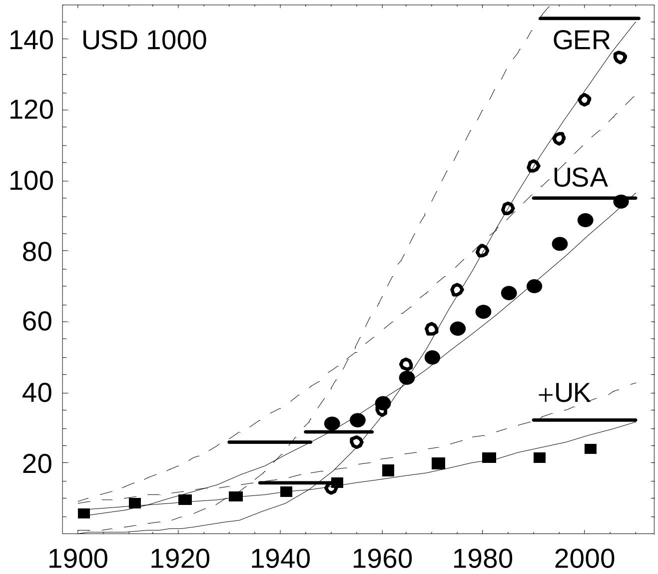

Figure 3: The time shifts between physical capital per capita (data and theory in US Dollars of 1991) and the normalized GDP (dashed from Figure 1) for West Germany, USA, and 19th century UK shifted into 20th century.

Using the identity $c\,e^{\gamma t} \equiv e^{\gamma(t+\Delta t)}$ with $\Delta t = (1/\gamma)\,Log\;c$, and inserting (3b) and its derivative $\dot{y}$ into (13) yields with some patience

(15) $$\mu = \bar{\mu}\, y_k\,/y$$

and from (9)

(16) $$k = \bar{\mu}\, G\, y_k\,.$$

The only difference between y_k and y is that the half times of y_k,

(17 a) $$T_k = T_a + E \cdot Log\,(1 + G/E) = 2061$$

and

(17 b) $$\tau_k = \tau_y + \beta^{-1} \cdot Log\,(1 + \beta G)$$

are simply delayed by 21 years for the evolution, and between 12 and 18 years for recovery. The bars over the year 2000 in Figure 3 span 20 years instead of the 21 they will measure in 2040 and beyond. The bars over the year 1940 span the theoretically required 13 years for Germany, 16 years for the USA, and 15 years for the early UK.

The early UK data are taken from [12], the other data from the national statistics. The dashed GDPs are identical with the plots in Figure 1, normalized to the final levels of $\bar{k} = \bar{\mu}\, G\, y_k$ with $\bar{\mu} =$ 0.18 for the UK and the USA, and 0.25 for West Germany. The UK's evolutionary data were shifted by one century into the time frame of Figure 3.

Figure 4 shows the capital share (15) and $100\dot{\mu}/\varepsilon$ instead of $\dot{\mu}\,G$ for West Germany.

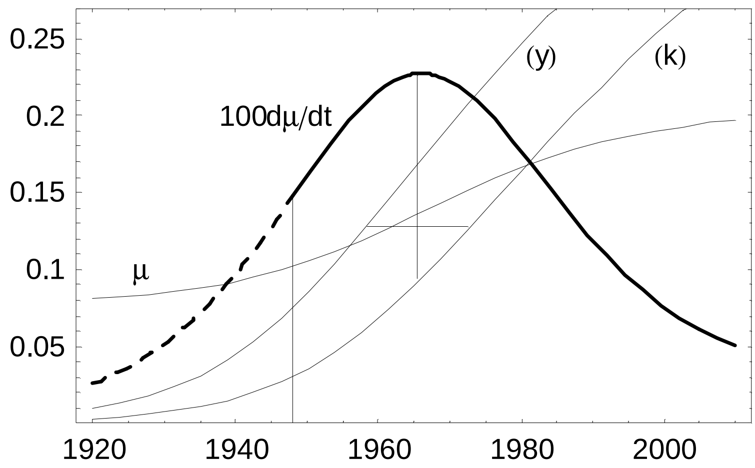

Figure 4: Comparison of the capital share μ and $\dot{\mu}$ in per cent p. a. with the normalized GDP and physical capital of West Germany. The theory applies after renormalization of the economy in 1948 with the DM as new currency.

Also shown is their position relative to *y* and *k*, both normalized to 1 on the vertical scale. The small contribution of the evolution *a(t)* to the capitalization term shifts the maximum of $\dot{\mu}$ by 6 years to the left of West Germany's halftime of recovery in 1971. The horizontal bar shows again the time shift (17b). It is clear that the exact theory can represent recoveries only for the entire range $-\infty < t < \infty$ although it can be valid only after the economy is renormalized for a stable recovery. West Germany's "Social Market Economy" was created in 1948.

Main result: The time shifts between *y* and *k* show that the exact theory is self-consistent.

8. The next financial crises and China's convergence

Many advisory groups were set up for making recommendations against reoccurrence of global financial crises. According to the Report of the High Level Group on Financial Supervision in the EU "the fundamental underlying factor [leading ultimately to the 2007 crisis] was the ample liquidity and related low interest rate conditions which prevailed globally since the mid-nineties, fuelling risk taking by investors, banks and other financial institutions" [13]. This means that the occurrence of such conditions is accepted as fact so that all proposals address only the symptoms.

The fundamental question is what force can cause a condition of surplus investment money, because knowing it is definitely preferable to waiting for the symptom which so far meant that the damage already happened. This force must obviously be stronger than the forces driving us into research, innovation, and investment, not to speak of the investments considered urgent by nearly every government. The superficial force is greed for high returns, a century ago already the motive for suggesting negative interest rates for idling money. But we have now an inescapable force which is stronger than any other, namely the biologic stabilization of the industrial evolution.

The agreement between data and theory for the recoveries in Figure 2 shows that the leading economies grew nearly linear with time from 1960 to date. According to (2b) linearity will continue through this century because of the evolution's symmetry to the year 2040 and the long evolution constant. This means with (3a), (16) and (17a, b) that the real per capita growth rate of physical capital

(18) $$\dot{k}/k = \dot{y}_k / y_k$$

decreases in this century for the leading nations inversely proportional to time. This is a hyperbolic decrease as shown with the dashed line $(dk/dt)/k$ in Figure 5 for Germany after 1975. Presently the long term growth rate is for leading nations $\dot{k}/k \cong 0.014$ p.a. per capita. According to (18) and Figure 5 it will drop to 0.005 p.a. at the end of this century.

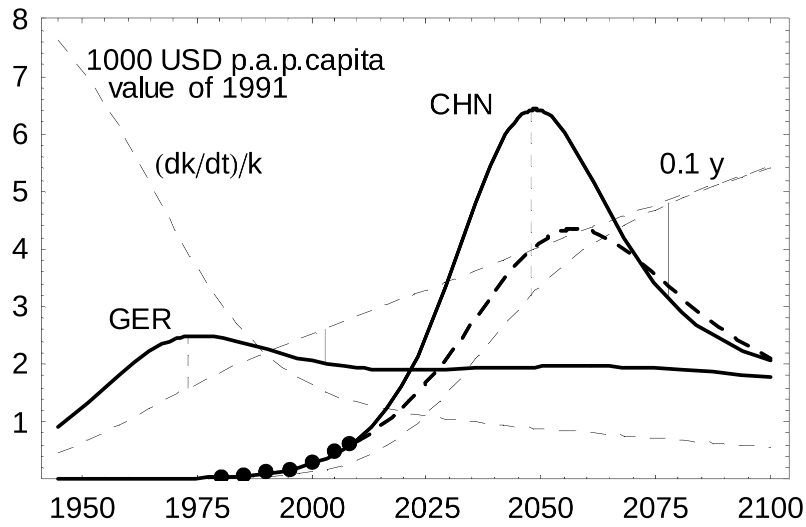

Figure 5: Prediction of annual investment in new physical capital for Germany and China compared with 10 % of their GDP per capita, China's controlled course (dashed), and the growth rate of Germany's physical capital in per cent.

Investors and banks expect however exponential returns on investment. This gap between expectation and reality causes the observed "cognitive dissonance crises". They are inescapable unless the financial system adjusts to the very small and decreasing growth rates for investment capital at the level of the industrial evolution. This will require a level of modesty and professionalism that can hardly be expected when an order of magnitude larger volume of speculative transactions takes place under the same roof. But knowing now the real demand for investment capital allows separating incompatible businesses clearly without nationalizing the banking system.

The second kind of new banking crises are the national "convergence crises". They are caused by the transition from the large growth rates of recovering or catching up into the small growth rate of the industrial evolution. We remember from Section 2 that the industrial evolution is slow because there are no superior societies to copy from. China's recovery is especially fast because of the government's early and strict joint venture policy. It included even the transfer of knowledge beyond the technology needed for the actual joint venture.

The first convergence crises were observed between 1980 and 1987 when West Germany and Japan experienced a sudden decrease of new investment and employment. Their cause was not understood because there was no idea of an inescapable hierarchy between recovery from disaster and the industrial evolution. The decreases were put up with an undefined "weakness of growth" in West Germany and a "hollowing out" in Japan, which had even a negative growth rate in 1983. Large sums of capital were injected. This is the recipe for damping negative phases of business cycles (Section 12), but it is the worst that can be done in a convergence crisis because we will see immediately that there is already a surplus of capital for investment when the gap to be closed in (3a) vanishes.

The forecasting power of exact solutions applies also to all integrals and derivatives. When for instance the data for *y* or *k* identify a stable $\overline{\mu}$, all variables derived from *y* or *k* such as μ, $\dot{y}$, $\dot{k}$, or $\ddot{k}$ etc. will automatically agree with their respective data. With this general rule we could identify Japan's real estate bubble from national reports. The data for *y(t)* disclosed via the real β and the theoretical $\overline{\mu}$ of (14) a systematic overvaluation of k by about half the real estate value of Tokyo. Forecasting quality includes forensic quality.

China has the same parameters *E*, *G*, and $\overline{a}$ as every nation for catching up and converging with the evolution. It is committed to recover, achieve convergence, and lead. Its GDP shows from 1980 to date a stable support for physical capital of $\overline{\mu} = 0.28$. The OECD predicts stabilization of the population at 1.6 billion. Halftime of the predicted GDP per capita will be in 2040, coinciding with that of the industrial evolution. When China maintains its integrity and has the favour of peace, it will converge in 50 years with the industrial evolution.

Figure 5 shows the annual addition $\dot{k} = \overline{\mu} G \dot{y}_k$ of physical capital according to (18). The vertical bars to the plots representing 10 % of the GDP meant for West Germany that $\dot{k}$ decreased by$15 - 7 = 8\%$ relative to the GDP. 3 Million people lost their jobs. A rough estimate with linear accumulation from 0 to 8% amounts in 25 years to a surplus of $0.04 \cdot 25 = 1$ full GDP of investment capital, in 1990 about 20.000 US$ per capita. Germany was very lucky with the collapse of the former Soviet Union because it opened in former East Germany within a narrow window of time just the missing demand for investment capital. A population ratio of 0.2 required just about 20.000 US$ per capita for a complete renewal of East Germany's physical capital to the West German level of 100.000 US$ per capita.

When 8 % troubled already Germany and Japan, China's loss of $18 - 7 = 11\%$ of its GDP for investment can mean that up to 60 Million people lose their jobs. For the global capital market it will be a disaster because China will practically own it. The surplus capital would accumulate to $0.055 \cdot 25 = 1.4$ GDPs per capita, or with a GDP of 42.000 US$ in 2060 and 1.6 Billion people $9.4 \cdot 10^{13}$US$ in their value of 1991. Section 15 shows that this amount corresponds to the world's total assets of 2007. This largest convergence crisis of all times can only be tempered with a controlled reduction of the total capital share used in production. When started immediately, the dashed course results with a reduction of $\overline{\mu}$ from 0.28 to 0.23. This reduces the convergence parameter from 0.10 to 0.075 p. a., and the depth of the crisis to the critical but survivable extent it had for Japan. It seems that China can use its surplus power for stabilizing the existential conditions in its inland provinces.

<u>Main results:</u> The banking system must adjust to S-functional growth. The industrial society's worst predicted convergence crisis can be tempered when China starts immediately with a controlled reduction of its productive total capital share in favour of improving the existential conditions in the rural provinces.

9. The inevitable trade-off between leisure and working time

National employment levels are stabilized near 1:1, not least because reproduction is the responsibility of pairs. This leaves working time as the only valid variable for labour. Since the distribution of unpaid work at home and paid work out of home is on average fair, paid and unpaid working times are comparable. Then the spare times for employed and unemployed are also comparable. This means that the trade-off (1) in Section 2 between working time for producing the GDP and spare time for using and hopefully enjoying the GDP is an essential part of economic equilibrium.

Industrial development shows that the design of physical capital determines from ploughs to hospitals the qualification as well as the annual working time required for operating physical capital. The required annual working time depends therefore on the current mix of actually built and operated physical capital. Working and spare time are therefore cultural variables like life expectancy and industrial evolution. Figure 6 shows this with the anti-correlation between the official weekly working time *w(t)* and *L(t)* or *a(t).* It also confirms the maximum annual working time of 96 hours per week introduced with (1) in Section 2.

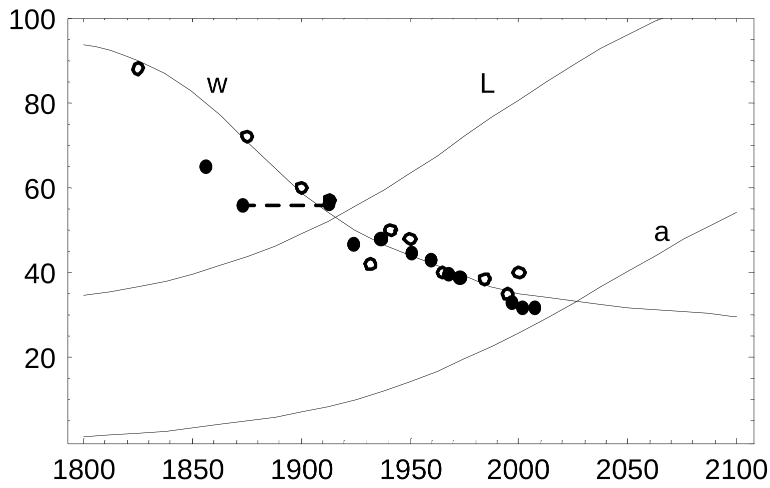

Figure 6: The cultural variables of the industrial society. L(t) in years and a(t) in US$ 1.000 p. a. p. capita from Figure 1. Official working time w(t) in hours/week for UK (points) and Germany (circles) compared with theory from Appendix A1

The plot *w(t)* is a programming exercise in Appendix A1 for the cultural transitions from agriculture, where work was still united with home and the village, to manufacturing, and now to the lap top office. *w(t)* is a cultural variable because it also shows hardly a trace of last century's disasters. Obviously the design rules for *k* are also paced by *E*. This can be understood when the technical subset of the relevant knowledge must be embodied in *h* prior to its design into *k*. As already mentioned with Figure 1, the dashed stagnation of the UK was due to the growing competition of Continental Europe and the USA.

Barro and Sala-i-Martin [14] gave a comprehensive quantitative account of the central role neoclassical production functions have since Solow's pioneering work. For mainstream theory they offered with the exponential function the first access to industrial dynamics. Neoclassical theory allowed also using the beautiful theory of partial derivatives and explaining the important phenomenon of diminishing returns. But designing it into the theory with adjustable exponents spoils the forecasting potential of exact mathematics, and the need for introducing fractional exponents shows that the production process cannot be really understood with the costs of work and physical capital.

The condition sine qua non for using the theory of partial derivatives is that the input variables are independent. Otherwise their dependence must replace the production function. Two more problems join in leading the way to the next section's replacement of mainstream production functions with an exact expression. In the long run the technical level is included in the value of physical capital because not even the manufacturer can separate them, and fractional exponents are and create non-existing quantities so that the relevance of results obtained with formally allowed mathematical operations is not guaranteed.

Main results: Mainstream production functions cannot describe dynamic processes and continuous equilibrium because labour is not an independent variable. Working and spare time are cultural variables like the life expectancy and the evolutionary GDP.

10. The equation of state for dynamic equilibrium

Physical capital is designed as amplifier of human working time. This means that the GDP is proportional to both *w* and *k*. But the symmetry (10) used for calibrating the initial values of the capacity and capital shares (11a, b) is lost when physical capital is duplicated fast with money, because it does not carry relevant knowledge, whereas increasing human capacity per capita needs time for learning and social interaction. Therefore the asymmetry would change faster in favour of producing the GDP than the cultural variables in Figure 5 plus spare time at home could follow.

The resulting disequilibrium becomes untenable unless an increasing part of physical capital is produced for public and private use instead for more production. This explains the increasing expenditures for housing and mobility. They improved the existential conditions but did not necessarily increase the spare time at home. When the part k_w remains for production, the GDP is given by

(19) $$y = w k_w .$$

This relation is exact, but it is not a true production function because *w* depends on *k* as shown in the preceding Section. The validity of (19) is confirmed by the agreement of *w(t)* from Appendix A1 with the data in Figure 6. The very simple form (19) is mainly due to the choice of $\bar{\varepsilon} = 1$ p. a. as unit for measuring working time. Then the capital coefficient $k_w / y = 1/w$ for production represents formally the number of years it would take to produce k_w with the same productivity of producing the GDP.

Since we want an exact expression for the equilibrium between all variables of production and use, we need a corresponding expression for life at home. When physical capital amplifies working time for producing the GDP, symmetry requires that human capacity amplifies spare time for using the GDP. Since $k_w < k$, the human capacity needed for the user side must also be smaller than the educated value following from (8), $h_s < h$. This yields with (1) and (19)

(22) $$(\bar{\varepsilon} - w)\, h_s = y = w\, k_w \,.$$
We call this pair of equations the "economic equation of state" because it defines the dynamic state of continuous equilibrium between all variables.

(2b), (3b), (16), and (22) are without any adjustable parameter valid results for times of peace, and the main predictions of the exact theory for economic growth for the full range $0 \le y \le \bar{a}$. An existential minimum for the reproductive minimum of the life expectancy is introduced in Appendix A1. Beyond that the main question is how much work people are willing or forced to accept for an increasing public sector and physical capital they cannot own. That the entire GDP must be taken for quantifying economic equilibrium instead of just the consumer goods was explained already for the symmetric equilibrium condition (10).

(22) can be resolved for the dependent variables, i. e. the annual working and spare times

(23a) $$w = \bar{\varepsilon}\, h_s \big/ (h_s + k_w)$$
and
(23b) $$s = \bar{\varepsilon} k_w \big/ (h_s + k_w)\,,$$
and for the exact production function
(24) $$y = \varepsilon h_s k_w \big/ (h_s + k_w)\,.$$
When k and k_w are produced with the same productivity, and the same is assumed for human capacity, we have
(25) $$k_w / k = h_s / h\,.$$
h_s and k_w follow from resolving (22) for the independent variables since w and y are known from national statistics. Whereas the value of physical capital can be exactly measured for every piece of equipment, the individual values of human capacity cannot be exactly measured. This gives capitalism a quantitative advantage over the other alternative where human capacity and spare time for life at home would be the primary quantities for the industrial society.

Section 6 showed that k runs parallel to y with the time shifts given by (17a, b). The same procedure yields
(26a) $$h = (\bar{h} / \bar{a})\, y_h$$
with
(26b) $$\bar{h} = \bar{a} / \varepsilon\, (1 - 1/\bar{\mu}\, \varepsilon\, G)$$
from (24). The difference between y_h and y are only the time shifts

(27a) $$T_h = T_a + E \cdot Log\,[1 - G / E\, (\bar{\mu} \varepsilon\, G - 1)]$$
and
(27b) $$\tau_h = \tau_y + (1/\beta)\, Log\,[1 - \beta\, G / (\bar{\mu} \varepsilon\, G - 1)]\,.$$

Both time shifts are negative, for Germany – 5 and – 6 years for (27a) and (27b). They compensate the positive time shifts (17) of k and k_w so that (24) and (3b) coincide.

Figure 7 combines all main values for Germany. The later entry of Germany compared to the

UK and its rapid recovery after WWII are seen in y, k_w, and also in h_s as explained for (22). When the disaster is short compared to E and education continues at the same relative level, h is unchanged because relevant knowledge per capita is indestructible. This is different for nations starting late with industrialization and education. Section 8 explained China's effective joint venture policy for solving both problems.

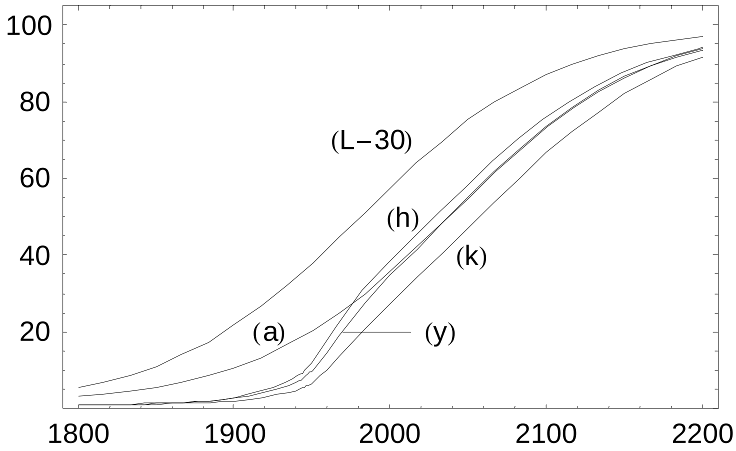

Figure 7: The normalized values of L-30 and a(t) with y, k_w, *and* h_s *from Appendix A1*

The proximity of h_s to y and consequently of $h(t)$ to $a(t)$ proves that h guides $L(t)$ and $a(t)$ nearly simultaneously.

Figure 8 and is caption offer a geometric explanation of the equation of state (22). The parametric data are from Figure 5 with the GDPs from Figure 1. Plot 1 would result when the full k would be used for production, plot 2 results for k_w according to (19), and plot 3 would result for full symmetry $k_o = h_o = 2y_o/\varepsilon$. The dashed line shows the existential minimum $\varepsilon h_m = y_m = \varepsilon k_m$ introduced in Appendix A1.

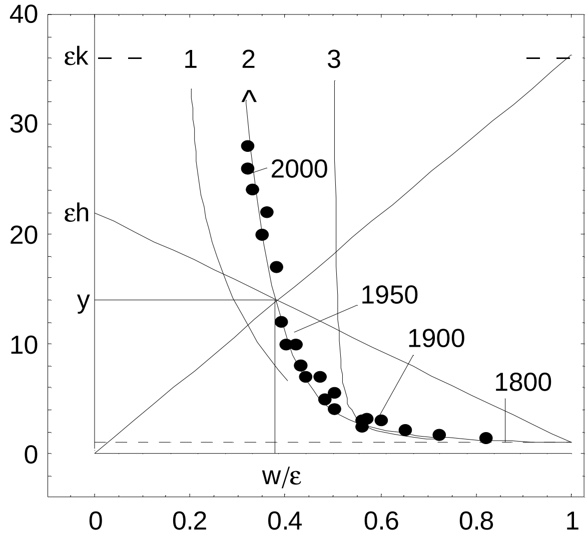

Figure 8: Geometric explanation of the equation of state. The state (h_s, k_y, *w, y) is given by the intersection of the straight lines for* εh_s *and* εk_y. *The scale is in 1.000 US\$ p.a. of 1991*

<u>Main results:</u> The equation of state expresses the continuous dynamic equilibrium between all main variables without adjustable parameters. Adam Smith's invisible hand is embodied in the human capacity of every individual.

11. The real cause of diminishing returns

We have seen that the S-functional shape of human capacity, mean life expectancy, and evolutionary GDP per capita is the inevitable consequence of biologic pace control and stabilization. After successful recovery this applies also to physical capital. The S-functions look as if the diminishing returns we know from mainstream theory would also exist by virtue of the exact theory, but there doesn't exist any quantity with an absolute diminishing return. There is only the human species' omnipotence limit and the corresponding diminishing gap to be closed for reaching the asymptotic limits $\bar{a}$ and $\bar{L}$.

The relative diminishing returns follow from the structure of (24): When one stored quantity is large compared to the other, the GDP is limited by the other. Tracing back the derivation of (24) discloses that the real cause of relative diminishing returns is the conservation of time expressed by (1): We have only one and the same time for using and producing the GDP. There is also no reason for assuming any non-linearity in the products of (22). The new theory is complete and exact without any adjustable parameter.

The inversion

(28) $$1/y = 1/\varepsilon h_s + 1/\varepsilon k_w$$

of (24) shows the relative diminishing returns directly. The larger a stored quantity, the smaller is its relative contribution to removing the obstacles for a better life. Alternatively one can think of projecting the user capacity onto the producer capital and vice versa because (28) is formally identical with the optical imaging formula for mirrors or lenses. The GDP corresponds to the focal length, but the decisive difference is the goal to increase *y* with the feedback loops (8) and (9). The best models are therefore flow diagrams with their perfectly known rules and device characteristics. In fact, simple water flow models of the economy were constructed in the UK and Germany shortly before WWII.

(22) to (24) and (28) are still symmetric and neutral with respect to the storable quantities. Differentiating (28) with respect to time yields

(29) $$\frac{\dot{y}}{y} = \frac{\dot{h}_s/h_s}{1+h_s/k_w} + \frac{\dot{k}_w/k_w}{1+k_w/h_s}.$$

It shows how the contributions to the GDP growth rate of an input's growth rate deceases with the increasing value of this input relative to the other. This confirms that human capacity and physical capital must be increased together for the optimum path.

<u>Main results:</u> Absolutely diminishing is only the gap to the human species' omnipotence limit. Relative diminishing returns between h_s and k_w are due to the fact that we must divide our time between producing and using the GDP.

12. An exact theory of business cycles with damping, loss, and risk

In Section 2 we found out that the economy has three dynamical responses where one can be oscillatory. More or less regular fluctuations are called business cycles. Their legitimacy is controversial. The situation is best described by the bandwidth ranging from accepting the physical oscillator equation [15] or the Lotka-Volterra equations [16] a priori to denying the legitimate existence of cycles because a propagator is missing [17]. This is actually justified because the mass and restoring force propagating cycles in physical oscillator systems are missing in flow systems. Then the oscillator equation cannot explain why cycles should exist at all. The decisive quantity for proving that a dynamic response is more than the economy's reaction to shocks is the product of damping constant and cycle period, but so far both have not been derived for a real economy because the period depends on the damping.

On the other hand the causes and control of unstable growth, unemployment, and inflation are central problems of the industrial society [18]. Their complexity, documented by many schools of thought, seems to exclude analytic solutions. Controlling unemployment, inflation, and growth is therefore based on rules of thumb and mathematical approximations. A typical example is the phenomenological relation $\Delta i = -(1...2)\Delta u$ for the percentage change between inflation and unemployment known as "Phillips Curve" [18].

Figure 9 shows the only existing example of 4 consecutive business cycles. Its 1965 wing corresponds to a Phillips Curve, the ideal but unsustainable growth phase of small inflation with increasing employment. Considering that this phase was exceptionally long because of the Vietnam War, the average period of cycles for the entire sequence are the biblical 7 years.

We use for Figure 9 the mirror image of the original in order to get the standard phase direction for polar coordinates. The added ellipse is analyzed for phase relations in Section 13. It shows the approximate range of regular cycles after correction of the large perturbations by the added arrows, explained in Section 14 as oil price shocks. In contrast to every cycle's specific Phillips Curve the axis direction of the ellipse averages over all cycles.

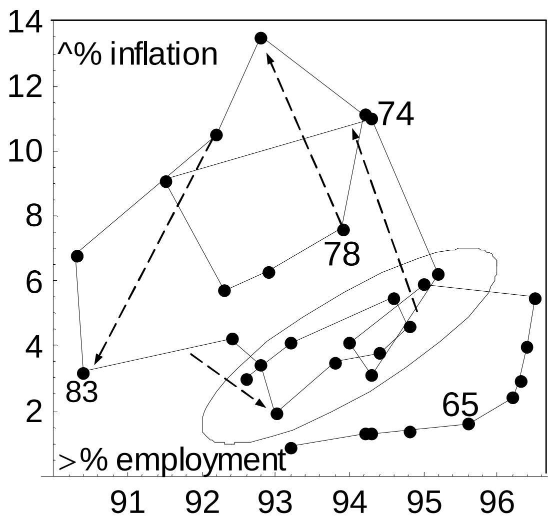

Figure 9: Four business cycles of the USA from 1961 to 1992 adapted from [18]. Arrows identify crude oil price shocks. The ellipse indicates the corrected range of cycles. The numbers in the plot are consecutive years from 1961 to 1992

The first oscillations in flow systems were observed in the 19th century when bookkeepers of the Hudson Bay Company analyzed synchronized fluctuations of the trapper's fur deliveries from hares and foxes. Ginzburg and Colyvan gave a survey of population cycles [19]. The possibility of cycles was demonstrated in 1925/26 with the Lotka-Volterra equations.

A better analyzed case of predator-prey oscillations are the relaxation oscillations of Nd:YAG lasers observed in 1970 between exited states (prey) and photons (predators) [20]. The oscillations cover the entire range from noise to full periodic clearance of excited states with high power photon spikes. Since the lifetimes of excited states and cavity photons are in the micro- and nanosecond range, the cycles could be examined within milliseconds instead of waiting for 30 years as with Figure 9. If their economic relevance had been known, the experiments would have been extended to study control of business cycles and shock resistance in detail. This is still recommended because now it could be done in many labs with much less effort.

A 7 year cycle period cannot be due to h and k because of their long reaction and life times of 62 and 25 years. The relevant quantities can only be consumer goods and the associated employment, income, and inflation. Since the sum of produced and consumed products with a phase shift between them cannot produce a clean signal, the pair employment and inflation is observed in Figure 9. The following theoretical detour via consumer goods is nevertheless necessary because a direct non-linear coupling between employment and inflation is difficult to see and formulate. On the other hand we need a non-linear coupling term between production and consumption because we can only set up a pair of linear rate equations for each subsystem, and linear systems cannot oscillate.

The non-linearity is caused by the consumer market because it is stimulated simultaneously by demand q and supply p. This means that the market exchange term is proportional to the product pq. We note that the equivalent product exists also in the equation of state (24), but there can be no oscillation because life and the accumulation of relevant knowledge are irreversibly locked to the passage of time.

We assume that all producers have a time horizon τ_p for producing p consumables so that the annual flow is p/τ_p. When the consumers have a time horizon τ_q for consuming q goods their consumption rate is q/τ_q. Then the consumption balance is given by the rate equation

(30) $$\dot{q} = pq/\bar{p}\tau_q - q/\tau_q \,.$$

We have used for simplicity the fact that a constant normalizing denominator $\bar{m}$ for the market exchange term $pq/\bar{m}$ yields for stable equilibrium $p = \bar{p}$ and $\dot{q} = 0$ the result $\bar{m} = \bar{p}\tau_q$.

As the Nd:YAG laser experiment and numeric solutions show, a non-linear market based on (30) and an equivalent equation for production can be totally cleared. An analytically closed solution for business cycles can however be obtained for small oscillations $q = \bar{q} + \delta q$ and $p = \bar{p} + \delta p$ so that the non-linear product term $\delta p \delta q/\bar{p}\tau_q$ can be neglected. Then (30) yields

(31) $$\delta\dot{q} = (\bar{q}/\bar{p}\tau_q)\delta p \,.$$

We assume also that the producers can adjust a fraction *x* of their production for anticipating changing demand, and that there will be a small loss rate p/τ_o in the distribution chain. Then the production balance is

(32) $$\dot{p} = \overline{p}/\tau_p + x\delta p/\tau_p - p/\tau_o - pq/\overline{p}\tau_q .$$

By inserting $p = \overline{p} + \delta p$, eliminating all *p*-dependences with (31) and its time derivative, and neglecting the product term $\delta p\delta q$, we obtain finally the oscillator equation

(33) $$\delta\ddot{q} + (1-x)\delta\dot{q}/\tau_p + (1-\tau_p/\tau_o)\delta q/\tau_p\tau_q = 0 .$$

Its damping constant, defined by the exponential decay $\delta q = \delta q_o e^{-t\cdot d}$ of cycles, is given by

(34) $$d = (1-x)/2\tau_p ,$$

and its circular base frequency by $\omega_o = (1-\tau_p/\tau_o)^{1/2}/(\tau_p\tau_q)^{1/2}$. With $\omega^2 = \omega_o^2 - d^2$ and $T = 2\pi/\omega$ the period of the oscillations is given by

(35a) $$T = 2\pi(\tau_p\tau_q)^{1/2}(1-\tau_p/\tau_o - (1-x)^2\tau_p/4\tau_q)^{-1/2} .$$

The observed period of 7 years means that the time horizon of producers and consumers is still the agricultural calendar year, $\tau_p = \tau_q = 1/\varepsilon$. Then (34) simplifies to

(35b) $$T = 2\pi\varepsilon^{-1}[1 - 1/\varepsilon\tau_o - (1-x)^2/4]^{-1/2}$$

With a loss rate of $1/\tau_o =$ 0.1 p.a. and a producer flexibility of $x = 0.5$ we obtain 6.9 years for the period and with (34) $Td = 1.7$ for the decisive damping-period product. This means that an oscillation propagates to the next cycle with an amplitude ratio of $e^{-T\cdot d} = 0.18$. This is just about enough to speak of an oscillation and explains why real business cycles are so rare. The sequence of 4 consecutive cycles in Figure 9 was assisted by three external events whose spacing was close to the cycle period (Section 14).

When shocks affect primarily the consumer market the damping constant is simultaneously the economy's reaction time for reaching a new equilibrium condition. Then the 1/e-reaction time is always given by $1/d = 2$ years because by definition a shock cannot be anticipated so that $x = 0$ in (34). This agrees according to Figure 11 of Section 14 with the 2-3 year precedence of the interest rate's return to normal before the first collapse of the oil price marked "SU" occurred. Figure 9 shows that unemployment follows about 1 year after an inflationary shock, and that employment follows about 1 year after the return of inflation to the normal level of the ellipse. Employment driven shocks lead in turn to 1 year delays of the inflation rates.

For a 10 % oscillation the producer must anticipate with $x = 0.5$ only 5% of his production correctly. His risk is relatively small, but he must know the phase which will be analyzed in the next section. But when he knows the phase as an insider, he wins whatever he anticipates.

<u>Main results:</u> Business cycles can in principle occur for consumer goods with the calendar year as planning horizon. This results in a base period of 2π years. Clear cycles are so rare because without anticipation the damping constant is 0.5 p.a. corresponding to a 1/e reaction time of exactly 2 years.

13. Structural and cyclic inflation

The four arrows in Figure 9 correspond to the oil price jumps identified in the next section. Their correction brings the data points inside the ellipse

(36a) $$e = e_o + \delta e \cos(\omega t)$$

and

(36b) $$\gamma = \gamma_o + \delta\gamma \cos(\omega t - \varphi) \qquad .$$

plotted in the unit of per cent with centers at $e_o = 94$ for employment and $\gamma_o = 4$ for inflation, the amplitudes $\delta e = 2$ and $\delta\gamma = 3$, and the phase shift $\varphi = \pi/4$. The time delay of inflation relative to employment is

(37) $$\Delta t = (\varphi / 2\pi)T$$

where $T = 2\pi/\omega = 7$ years is the mean cycle period from the preceding section. That inflation follows employment within $\Delta t = T/8 = 0.9 \pm 0.1$ years confirms the estimated reaction time for salaries [18]. The accuracy follows from the sensitivity of the axis ratio to a change of φ. It is better than expected with a one year resolution of annual statistics since the ellipse averages over four cycles. The desirable boom phase of the Philips Curve corresponds for the amplitude ratio $\delta\gamma/\delta e = 3/2$ of the ellipse (36a, b) to the phase range $\omega t = (1.3...1.7)\pi$.

In order to avoid inflation and deflation the central banks try to keep the ratio $\lambda = \varepsilon m / y$ between the money supply m and the real GDP constant. The inflation rate

(38) $$\gamma \equiv \dot{\lambda} / \lambda = \dot{m} / m - \dot{y} / y$$

vanishes when the money supply rate equals the real growth rate. With increasing employment and GDP the money supply is increased. Their combination leads to inflation. The following negative phase is fought by injecting money again which is generally not removed in the following boom. The long term result is that the mean value of the amplitude $\delta\gamma = 0.03$ p. a. of (36b) tends to be the cyclic contribution to the long term inflation rate γ_o. The remaining question is the cause of the nearly constant shift $\gamma_o - \delta\gamma$ of the ellipse on the vertical axis in Figure 9. The condition $m\varepsilon = \lambda y$ which led to the relation (38) is formally identical with (9), the maintenance condition $k / G = \mu y$ for physical capital. Inspecting the total annual support (12) for k shows that the annual addition $\dot{\mu}\varepsilon G y$ to the GDP is not covered by $\dot{y}$. This means that the long term inflation rate is given by the ratio of this new term to the GDP,

(39) $$\gamma_o - \delta\gamma = \dot{\mu}\varepsilon G \, .$$

We call this inevitable contribution “structural inflation” because is due to the structural change of an economy when its capital coefficient k / y is changed. It is plotted in Figure 10 at the bottom for the USA and agrees with the ellipse’s base shift observed with Figure 9.

The West German data in Figure 10 are a superposition of three contributions. The large fluctuations correspond to the oil price shocks shown in Figure 11 for the USA. The dashed plot is West Germany’s structural inflation rate $\dot{\mu}\varepsilon G$ following from $\mu(t)$ of Figure 4. When it is corrected for the total lack of any returns from investments made during WWII, one obtains the base line plotted within the fluctuating data. This means that for both economies the capitalization term in (12) results fully in structural inflation. This is the consequence of duplicating physical capital just with money, i. e., without additional knowledge.

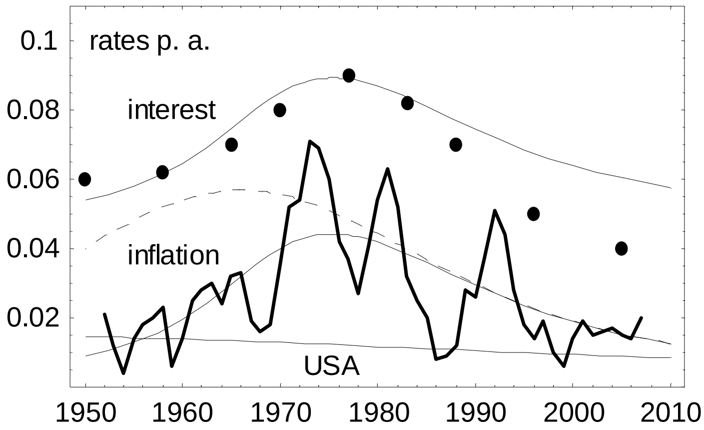

Figure 10: Structural inflation and interest (points) in West Germany compared with theory (plots) and the structural inflation of the USA

Since only the principal must be paid back after the agreed period the banks must include inflation rates in their interest rates. The upper plot is the sum of West Germany's inflation rate and a 4.5 % net interest rate. The data are the reported intermediate term interest rates. After 1985 the German banks reduced their interest rates due to the global surplus of investment capital after the convergence crises of Germany and Japan disclosed in Section 8.

14. Oil price shocks and China's energy demand

Figure 11 compares the crude oil price with the sum of the inflation and unemployment percentages of Figure 9. The unit of one third of a barrel for oil and a base line shift for inflation were chosen to emphasize the conclusion that oil price shocks caused the large excursions from the regular range of business cycles.

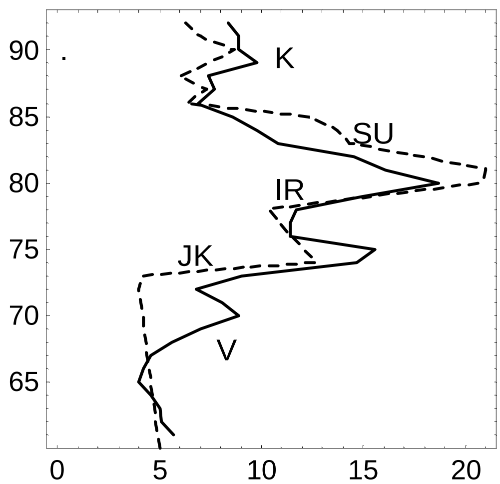

Figure 11: Comparison of crude oil price (dashed) in current US Dollars for 53 liters with inflation rates in % from Figure 9. The vertical axis is the time from 1961 to 1992

JK and *IR* stand for the oil price jumps due to the Jan Kippur embargo and the Iranian Revolution. From 1973 to the 1981 peak the total cost of crude oil had increased to over 60 US Dollars per barrel corresponding to 187 US$ to date with 4% average inflation p.a. For the USA it meant an increase from originally 2 to 9 per cent of the GDP, an extremely steep and large shock. The governmental subsidy during the *JK* jump was not successful enough to be repeated for the *IR* jump. *SU* denotes the drop of the crude oil price due to the collapse of the former Soviet Union. The smaller peaks *K* and *V* were due to the Kuwait and Vietnam War. The net result of the liberal inflation policy of the USA was that the OPEC countries got in real value less for their oil than before the price hike.

The preceding sections showed that the industrial evolution was from its beginning paced and limited by human biology. The first challenge to this limit will probably be our fossil energy reserves. Directly associated is the climate sensitive CO_2 release and indirectly the fresh water shortage, because it will require desalination of sea water and transport over large distance, which reduces in the end also to the energy problem.

Figure 12 shows the total flow of primary energy for the nations at the evolutionary level, for China, and the sum of both. Assumed are for the former a stable population of a billion people with the present flow of 6 kW per capita, growing proportional to the GDP (2b). The assumption for China is the OECD's projection of population growth from 1.3 to a stable 1.6 billion people and the same proportionality of demand growing according to (3b) with $\bar{\mu} = 0.28$ and $\beta = 0.10$ p. a.

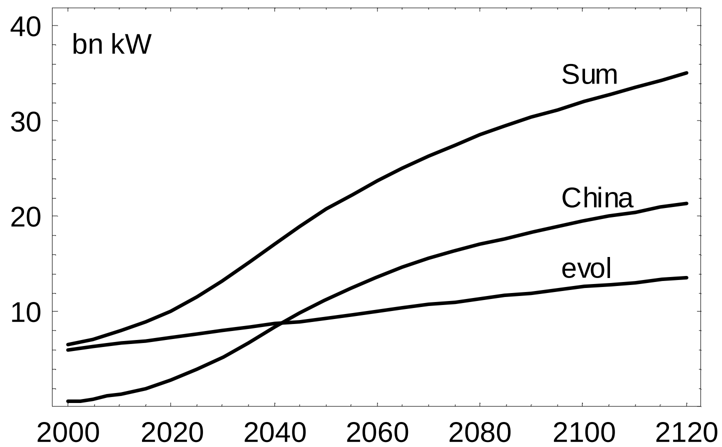

Figure 12: Prediction of the flow of primary energy for 1 billion people at the evolutionary level, 1.6 bn converging Chinese, and the sum in billion kilowatts

The average of $2.5 \cdot 10^{10}$ kW for the next 100 years will be increased by the rest of the world to about $3.5 \cdot 10^{10}$ kW. This is not a problem compared to the Gulf Stream which transports about $1.4 \cdot 10^{12}$ kW into the North Atlantic. But we burn already now every year a quantity of fossil energy which took 1 to 0.5 billion years ago the sun and life on earth up to 1 million years to produce. That this million-fold CO_2 release cannot leave our climate unchanged is clear without having to believe in the results of complex climate studies.

The present world consumption is with $6 \cdot 10^{13}$ kWh p.a. only slightly larger than the sum in Figure 11, and 90 % of it is obtained from burning $3.3 \cdot 10^{10}$ barrels (159 litres) p. a. of oil. The safe reserves are estimated to last for 50-60 years for constant consumption. Even when China continues to use mainly coal, Figure 11 means that already the children of the affluent nations will see the return to coal and railroads.

Crude oil is our most precious resource for organic chemistry from fertilizers, fibres, paints and plastics to construction materials. Just burning it for heating, air conditioning, and mobility is the most uneconomical human act imaginable after a third world war in terms of the cost to global resources. The problems discovered with the first net output reactor ITER show that nuclear fusion, the hope for safe and unlimited energy supply, will come too late for saving oil. Our presently most rewarding economical alternative is building as fast as possible thermo-power plants in the desert for generating electricity, fuel, and fresh water, the latter for the people owning the land and working there.

Main results: The first problem of staying at the level of the evolution will be the energy supply. Industry must urgently find a safe alternative for gas and oil. Preferable to burning coal or every other big effort is developing and building thermopower plants in the deserts.

15. Evolutionary pension funds

According to Figure 3 the life expectancy reached in the leading nations the traditional retirement age of 65 between 1950 and 1960. Already then it was clear that tax based pension systems with constant retirement age must collapse sooner or later. With a constant employment level, an average working life of 43 years, and a maximum pension time of 118 – 65 = 43 years, half of the income would be needed just for pensions with a relatively low pension level of half the working time income.

In the following we analyze the other extreme of one collective pension fund for everyone with a total volume $k_p(t)$. Its annual return is given by the effective interest rate $z(t)$. The pension level σ is given by the ratio of the average pension to the current GDP *y(t)* per capita. We will have a relatively low pension level when we assume that every individual above retirement age gets his or her own retirement income. We also assume that the fund pays at least the existential minimum to everyone after retirement age so that there is no need for maintaining any tax redistribution system for retirement.

The fraction p of pensioners is given by the ratio of two integrals, one over the tail of the age distribution above the retirement age *R*, and the integral over the entire distribution. In order to understand the functional dependencies between all variables we simplify the age distribution with an S-function decreasing symmetrically to the mean life expectancy with a decay parameter $D \approx 10$ years. This results in a $1/e^2$ tail width of 40 years. The integrals are equivalent to (5) and (6) with the results

(40a) $$p_{60} \cong 0.49/(1+e^{(1992-t)/48})$$

and

(40b) $$p_{85} \cong 0.28/(1+e^{(1965-t)/40}),$$

when the retirement age is increased from 60 to 85 parallel to the life expectancy (6).

With these parameters the required level of the fund is given by the balance

(41) $$zk_p = \sigma p\, y\,.$$

Comparison with (9) shows that the fund's balance corresponds to the maintenance balance of physical capital with $\mu \equiv \sigma p / z G$ and a total capital share of $\bar{\mu} \equiv \sigma \bar{p} / \bar{z} G$.

Figure 13 shows the required assets of three funds. With a stable interest rate of $\bar{z} = 0.04$ p. a. above inflation and $\bar{p} = 0.49$ or 0.28 from (40a) or (40b) we get for $\sigma = 0.4$ as result $\bar{\mu} = 0.20$ or 0.11, respectively. This means that the fund must own for the USA ($\bar{\mu} = 0.18$) more than its entire physical capital for retirement at 60, and over half of it when the retirement age is increased to 85 according to (40b). For Germany and Japan the corresponding levels are 75 % or 40% of their physical capital. The dashed fund level would be needed for the 85 fund with an interest rate of only 3 % p. a.

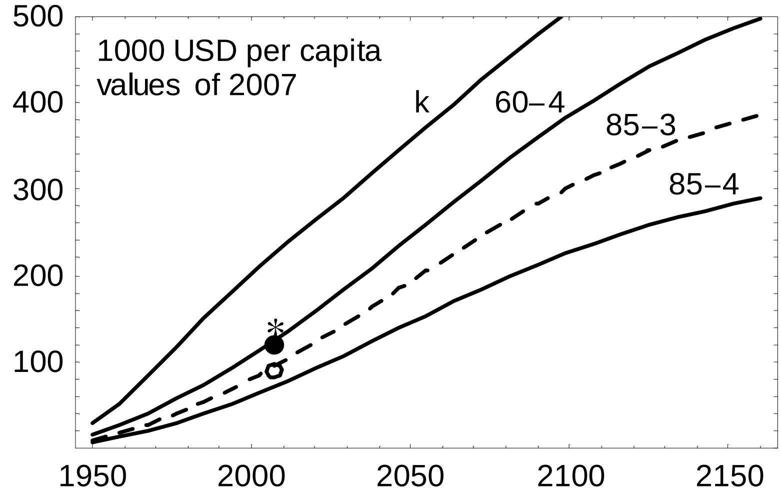

Figure 13: The required assets of 3 pension funds paying from 4 or 3% net interest 40% of the GDP per pensioner for retirement at 60 or growing to 85. For comparison Germany's physical capital k, *total wealth of Germany (circle) and the USA (point), and the level of total global assets when 1 bn people at the evolutionary level had equal access to it (star).*

Recently the "Issing Report" [21] estimated the amount of global assets in 2007 and their distribution. In units of 10^{13} US\$ their levels are 5.5 in the top 1.000 banks, 4.4 in mutual funds, 2.9 in pension funds, and 1.8 in insurance companies, in total US\$ $14.6 \cdot 10^{13}$. The star in Figure 13 results when the same 10^9 people consuming at the evolutionary level 6 kW in Figure 12 could equally share this total asset pension fund. Comparing this with the present pension fund level of $2.9 \cdot 10^{13}$ US\$ means that the latter covers less than 20% of the pensioners, provided they had equal access to the fund. But this is not the case for 10^9 people because existing pension funds serve only a fraction of the people at the evolutionary level.

The overall result is that pension funds would just about satisfy the minimum demands for the nations at the evolutionary level when their volume is comparable to the entire global assets. Minimum means a pension level of 40% of the current GDP, increasing mean retirement age to 85, and a minimum net interest rate of 3% p.a.

Aside from the fact that private saving for retirement runs the risk of total loss in case of war or financial collapse, we see that the other extreme of one collective pension fund requires a nearly flat distribution of wealth. But except for Japan less than 15 % of the population own 85 % of the national wealth. The third problem of pension funds is that due to the small growth rates after convergence an interest rate of 4 % above inflation can hardly be expected, especially after China joined the evolutionary level.

Main results: Uneven distribution of wealth and small evolutionary interest rates are serious obstacles for privatizing provisions for retirement. In this century a mandatory retirement age of 65 should be increased by one year for every 5 years. New concepts are needed for a fair distribution of working time.

16. Using the theory

This theory predicts the mean life expectancy, real GDP per capita, physical capital, its component needed for production, cost of maintaining them, annual investment, their growth rates and integrals, cyclic and structural inflation, annual working time, and required natural resources for every nation and region in times of peace. Human capacity, its component needed for sensibly using the GDP, and the minimum cost of formal education follow from the equation of state. The time dependences of all quantities are given by continuously valid, analytically closed solutions of three differential equations. They express the human species' talent for copying relevant knowledge from each other and ultimately from nature with research. The entire set of required parameters reduces to 3 constants of the human species and one national constant. There is no adjustable parameter.

National and regional diversity depends essentially only on Solow's saving constant $\overline{\mu}$. This allows an effective and fast diagnosis of economic states for comparison and prediction. The series of convergences shows for instance that Figure 1 opens under the industrial evolution a "field" for the GDP where every point (*y*, *t*) has its optimal growth rate and slope $\dot{y}$ according to (3a). The fastest way for identifying the economic state is plotting (3b), and adjusting the half time τ_y and the parameter of convergence β for agreement with the inflation corrected actual GDP per capita. The time for conversion of the exchange rates is 2007 for Figure 13, and 1991 for every other figure showing monetary values. From β follow $\overline{\mu}$, μ, and physical capital.

The corresponding procedure applies to μ and $\dot{\mu}$ in Figure 2 , *k* and $\dot{k}$ in Figure 3 and 5, annual working time in Figures 6 and 8, and inflation and interest rates in Figure 10. Every agreement with an exact solution is a valid prediction of the nation's development. The self-consistency of the theory means that when agreement is satisfactory for one variable, agreement will be found for every other variable. Disagreement points to incorrect data or even corruption, often indicated with fluctuating inflation rates and saving constants.

Non-negligible population growth rates $\dot{\rho}/\rho$ can be taken into account by replacing the generation constant *G* with

(42) $$G_p \cong G/(1+G\dot{\rho}/\rho) .$$

It means that a population growth rate of 4 % p. a. doubles already the cost of just maintaining the per capita value of physical capital. Such a high population growth rate all but excludes the possibility of real economic growth, i. e., for improving the population's quality of life.

Main results: The figures provide the fastest route to quantify and predict national and regional developments and to compare them with the optimum path of the exact theory. Population growth can easily prevent real growth.

17. Discussion

It was always clear that exponential growth cannot be the true growth law because nothing can grow ad infinitum for use by a finite entity, be it an individual or our planet. Therefore the only surprise of the exact theory can be its definite proof that human biology controls economic growth. That the S-functional growth law became observable only now is due to the fact that the transition from exponential to nearly linear growth happens according to (3b) within a few years around $\tau - 2/\beta$. With $\tau = 1971$ and $\beta = 0.090$ p. a. this time coincided, as indicated in Figure 4, with West Germany's and Japan's economic renormalization in 1948 and 1949.

The corresponding time for the USA is with 1925 close enough to the Wall Street Crash in 1928 for suspecting at least some correlation. The corresponding time for UK's evolution is $T_a - 2E =$ 1916 which fits the stagnation period observable in Figure 1, but we would not stress this point beyond its general relevance for the banking crises discussed in Section 8.

The proof of human biology's control of economic growth is given fivefold with the following discoveries and their quantitative explanations:

> All successful recoveries converge into the industrial evolution. This is shown with Figure 1 and quantified with (3b).
> The life expectancy parallels the industrial evolution (2b). The bio-economic relation (7) shows that their time shift equals half the mean final life expectancy.
> (3b) solves simultaneously Solow's generalized differential equation (12) for duplicating physical capital with money. Also significant for biologic control is that capitalism's stronghold (12), inflation, and interest are the only occasions where money was indispensable for the exact theory.
> Working time is a cultural variable like the industrial evolution and the life expectancy. This is shown with Figures 6 and 8, and quantified in Appendix A1.
> Every parameter of economic growth reduces to three constants of the human species *E*, *G*, $\bar{a}$ or $\bar{L}$, and one national constant $\bar{\mu}$. That the latter is determined by what people are willing to save in competition with existential goods and services reduces also to the biologically dominated question of the best possible life.

The frame consisting of these 4 constants, 3 differential equations, and the equation of state provides economic stability, forecasting power, and nearly perfect agreement with the observed time dependences of a, L, h_s, k, k_w, μ, w, y, and their derivatives in 4 leading nations with different cultural backgrounds. All problems left or originating from earlier theories are quantitatively explained or simply dissolve in the frame of the exact theory.

It is possible that the biologically stabilized constants $\bar{L}, \bar{a}, E$ and G change slowly at a rate small compared to $\pm 1/E$ p.a., because that cannot be detected within the short history of the industrial evolution. A serious resource shortage will not necessarily change these constants because one region or nation of sufficient size and power can perpetuate the industrial evolution.

The controversy over endogenous or exogenous causes of economic growth was fuelled by the contradictions between technical progress as exogenous cause, the exponential function's lack of an external driving force, and the phenomenon of diminishing returns. Paul Krugman found that increasing returns are possible with interregional trade [22]. This may be extended to include sectional advances like that of information technology, prematurely celebrated as "new economy" before China began to clear the main markets of the "old economy". Paul Lucas and Robert Romer found that non-competitive knowledge stored in educational reserves or still unused prototypes [2, 3] play a decisive role. We identified them quantitatively in Section 10 with $h - h_S$ and $k - k_W$, but they are not required for the exact production function (24) because there are no fractional exponents since diminishing returns are due to the trade-off between working and spare time. They had caused the search for hidden or wider definitions of input values as discussed by Robert Barrow and Xavier Sala-i-Martin for *k* [14]. The opposite problem of $k_w < k$ is solved in Appendix A1.

As every natural scientist and engineer can testify, the ultimate driver of economic growth is new knowledge obtained from nature with curious research, i. e. exogenous. On the other hand we have with the generalizing term $\dot{\mu} G y$ of Robert Solow's differential equation for duplicating physical capital (12) a relevant endogenous contribution to economic growth. This term is the basis of capitalism because it increases physical capital beyond proportionality with relevant knowledge, human capacity, and the evolutionary GDP. It causes also the full structural inflation rate $\dot{\mu} \varepsilon G$ shown in Figure 10.

Since spare time is given by (1), the only new quantity of this theory is human capacity *h(t)* per capita, the individually distributed carrier of the industrial society's relevant knowledge. Its part h_S required for economic equilibrium is exactly quantified with (22) by GDP and working time. The individual values of *h* cannot be exactly measured. The main reason is that *h* includes decisive heritable components which have no economic value because they consist of non-payable endowments. They provide the human species' enormous capacity to adjust even to extreme existential conditions. Only the S-functional shape and the mean values of *h* and h_S are exactly defined in this theory. But these properties qualify *h* as Adam Smith's invisible hand for maintaining economic equilibrium. This is probably as far as one can get with the present level of knowledge about human capacity.

The theory has other attractive characteristics. Already our undergraduate students can derive the economy's essential dynamic properties within a self-consistent frame of very few differential equations and constant parameters, everyone with a clear meaning for human life. The equation of state reveals for the first time the intrinsic beauty of the economy with its simple symmetry and conservation of time for life and work. The theoretical gap between economics and the natural sciences is closing because the relevant knowledge stored in human capacity is not separated into academic disciplines.

18. Outlook

The immediate future depends on how fast the financial disaster will be overcome. Since such events cannot be officially anticipated, (34) in Section 12 shows with $d = 0.5$ that the shock was overcome in late 2009. The recovery time Δt depends then on the nation's position along the growth path (3a) and the amount of public debt Δm accumulated for surviving the disaster relative to the annual value y/ε of the GDP. Assuming to first order again a linear relationship one obtains

(43) $$\Delta t = (\varepsilon \Delta m/y)/(\dot{y}/y) .$$

It will be very difficult for nations at the evolutionary level to increase their present per capita growth rate even temporally above the long term average of $\dot{y}/y \cong \dot{a}/a = 0.011$ p.a. Then (43) predicts for a public debt ratio of $\varepsilon \Delta m/y = 0.1$ and a 50% public share of the economy a recovery time of 20 years, provided the tax redistribution system remains unchanged. Since China's growth rate is $\dot{y}/y = 0.09$ p.a. the disaster left only a small and short dent in the GDP. This means that trade with China helps nations at the evolutionary level to recover fast.

For the long term future the theory predicts an industrial evolution between mainstream's exponential growth and the Club of Rome's original environmental feedback scenarios. We refrained from including the latter because the fossil reserves will probably not be known sufficiently well before the situation is undeniably critical. The first consequence will be a reduction of $\overline{\mu}$ mainly for the share $1 - k_w / k$ of physical capital for housing and mobility. This can be easily included in the theory as shown in Figure 5 with the example of China's convergence crisis. Physical capital and the GDP, but not necessarily working time, will smoothly decrease because all other parameters are constants of the human species.

Since 1.6 billion Chinese will probably get the problems of nations at the evolutionary level faster than the latter can solve them, solutions can and must be found together. By 2050 about half of the Chinese population will have joined Europe, North America, and Japan at the evolutionary level. Experience has shown that a strong middle class is needed as backbone of social stability and confidence in the future. This means however that the present trend towards increasingly uneven distribution of wealth must be reversed because the middle class cannot survive when it is mainly working for the rich and the poor.

Crude oil is the industrial society's most precious resource. Simply burning it for energy is after war the worst offence against the industrial evolution. The alternative recommended in Section 14 is to boost the construction of thermopower plants in the deserts for simultaneous production of fuel, electricity, and fresh water.

Section 3 showed that without a 3rd world war or another financial disaster the evolutionary GDP per capita will grow until 2050 to US$ 41.000 per capita in the value of 1991, US$ 15.000 or 58% more than in 2000. This means smooth and safe growth, so there is no point in making hectic and therefore unsustainable decisions. But economic growth will be very modest compared to the order of magnitude growth of the leading nations in the first 50 years after WWII. The banking, insurance, health and pension systems must adjust to this modesty. It makes no sense to fight a biologic stabilization which is to be expected from a species at the end of the food chain guided by natural science.

Acknowledgements, critique, and suggestions

We thank G. Metakides, the designer and director of the EU Commission's ESPRIT program, for his support of the only joint EU-MITI project to our knowledge. It started a discussion among CTOs of global companies about future markets and strategies, and resulted in the report [6] published with Y. Takeda, former CTO of Hitachi and head of the Japanese team. The required contents of the Japanese educational report [10] were translated by S. Kuwabara of the Japanese-German Center Berlin. Ron Kay of IMB/UC Berkeley treated us with challenging questions from 1978 to this text. Erik Aslaksen proposed the comparison with flow diagrams. The German Physical Society's Division of Socioeconomic Systems provided a congenial forum for discussion since 2002.

We thank the following colleagues for encouragement and constructive criticism: Paul Samuelson for putting us on track with his answer to our question "what unites economics as academic discipline?"; Mancur Olson for explaining to us shortly before his death his unpublished ideas on the convergence of GDPs with that of the USA; Robert Solow for his encouragement to publish our first results; Klaus Brockhoff for his hospitality during our first course on this theory at the University of Kiel; and the following colleagues for discussion and access to their libraries: H. Hanusch and A. Pyka, Augsburg; T. Krebs, Mannheim; H-W. Sinn, ifo and LMU München; D. N. Snower, ifw and Univerity of Kiel; and U. Witt, MPI and University of Jena.

In advance we thank all colleagues sending us their critique and suggestions. We encourage application of our theory to every nation and region, especially to Australia, Brazil, China, India, South Africa, and to the convergence of the European Union's member states. We would also like to know wherever there is an interest and the possibility of repeating the experimental flow models for comparison with our theory. They allow reducing the observation time for all phenomena by 5 and 8 orders of magnitude for the evolution (p. 23) and for business cycles (p. 25), respectively, and to test socioeconomic policies without risk and negligible cost. Please send your reactions to Danielmeyer@inb.uni-luebeck.de.

A1. Programming working time from 1800 to date and beyond

All graphics were programmed in Mathematica. This example programs the transition from agriculture to industry for the annual working time shown in Figure 6. It should be written line by line because Mathematica has its own indents and error warnings.

```
pnts=Show[Graphics[{Thickness[0.006], Circle[{1825,88},{2,1.3}],
Circle[{1875,72},{2,1.3}],Circle[{1900,60},{2,1.3}],Circle[{1913,57},{2,1.3}],
Circle[{1932,42},{2,1.3}],Circle[{1941,50},{2,1.3}],Circle[{1950,48},{2,1.3}],
Circle[{1965,40},{2,1.3}],Circle[{1984,38.5},{2,1.3}],Circle[{1995,35},{2,1.3}],
Circle[{2000,40},{2,1.3}],Dashing[{0.018,0.018}],Line[{{1873,56},{1913,56}}],
Point[{1856,65}],Point[{1873,56}],Point[{1913,56.4}],Point[{1924,47}],
Point[{1937,48}],Point[{1951,45}],Point[{1960,43}],Point[{1968,40}],
Point[{1973,39}],Point[{1997,33}],Point[{2002,32}],Point[{2007,32}],
Text[FontForm["working time/week",{"Arial",16}],{1840,90},{-1,0}]}]]
gdp=1+75/((1+Exp[(t-1948)])(1+Exp[0.016(2040-t)]+Exp[0.04(2000-t)]))+
    75/((1+Exp[(1948-t)])(1+Exp[0.016(2040-t)]+Exp[0.09(1971-t)]))
red=1-0.42/(1+Exp[(1955-t)/10])
kap=1+305/((1+Exp[(t-1948)])(1+Exp[0.016(2061-t)]+Exp[0.04(2017-t)]))+
   468red/((1+Exp[(1948-t)])(1+Exp[0.016(2061-t)]+Exp[0.09(1983-t)]))
wrk=96gdp/kap
curv=Plot[{wrk},{t,1800,2100},PlotRange→{0,100},Frame→True,
PlotStyle→{Thickness[0.004]}]
Show[curv,pnts,Prolog→AbsolutePointSize[8],DefaultFont→{"Arial",16}]
```

Pressing shift and enter at the end of the Show line produces the plot within a second.

"gdp" consists of 3 terms. The first is $a_o = 1{,}000$ US$ p.a., the assumed agricultural and existential minimum level of the GDP before industrialization. The second term is the GDP of Germany before WWII, vanishing with the S-function S_- represented by the first bracket in the denominator. National statistical reporting was suspended in the years 1942- 48. The third term is the GDP of West Germany after WWII, renormalized with S_+ represented by the first bracket in the denominator.

"kap" contains the corresponding 3 terms. Its initial value $a_o/\varepsilon = 1{,}000$ USD reproduces the full working time $w = \bar{\varepsilon}$ before industry separated work from home and *k* from *h*. This point is seen in Figure 8 as the starting point for all human capacity lines εh_s. The second term is Germany's physical capital before WWII according to (18) with $\bar{\mu} = 0.16$ and S_-. The third term is the physical capital of West Germany with S_+ and the S-function "red".

"red" describes the cultural change after WWII where according to the equation of state (22) an increasing part $(k - k_w)/k$ of physical capital was produced for life at home. It reduced the total capital share left for production from $\bar{\mu} = 0.25$ to $\bar{\mu}_w = 0.16$. The transition time of 10 years is the best fit to the data in Figure 8. It means that this cultural change happened essentially within the post war reproduction cycle.

A2. List of quantities

Biologically stabilized constants

$\overline{a}$ = US \$ 75.000 p. a., asymptotic maximum GDP per capita for $L \to \overline{L}$
$E = 62$ years, evolution constant, social coherence time of the relevant knowledge in h
$G = 25$ years, generation constant, mean physical lifetime of k
$\overline{L} = 118$ years, asymptotic mean maximum life expectancy
$L_o = 30$ years, mean minimum life expectancy for socially successful reproduction

Other constants

$\varepsilon = 1$ p. a., conversion factor between annual and storable quantities
$\overline{\varepsilon} = 1$ p. a., unit for measuring annual working time w
$\mu_o = 2/\varepsilon G = 0.080$, symmetric capital share
$\overline{\mu}$, total capital share, saving constant for k
$\nu_o = 2/\varepsilon E = 0.032$, symmetric capacity share
$T_a = 2040$, halftime of the industrial evolution

Long term Variables

a, industrial evolution, evolutionary real GDP per capita
$\beta = \varepsilon\overline{\mu}/2 - 1/G$ p. a., initial growth rate, convergence parameter into a
γ, inflation rate
h, human capacity per capita
h_s, part of h amplifying spare time for using the GDP at home
k, physical capital per capita
k_w, part of k amplifying working time for producing the GDP
L, mean unisex life expectancy of leading industrial nations
μ, capital share of the GDP for maintaining the current level of k
$s = \overline{\varepsilon} - w$, spare time for using the GDP at home
w, annual working time for producing the GDP
y, real gross domestic product per capita, converging into a

Cyclical Variables

$d = (1-x)/2\tau_p$, damping constant of business cycles
e, employment
$\varphi = \pi/4$, phase shift between e and γ_c
p, q, production and consumption value of consumer goods
$T(d, \tau_p, \tau_q) \geq 2\pi/\varepsilon$, cycle period
τ_p, τ_q, theoretically indefinite time horizons, practically still the calendar year
x, adjustable fraction of production

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