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The Intrinsic Dimension of Multispectral Images

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Abstract. In this theoretical paper we address the question of how to encode the local signal variation of multidimensional, multispectral signals. To this end, we first extend the concept of intrinsic dimension to the case of multispectral images in a way which is not depending upon the chosen colour space. We then show how additive, multiplicative, and occluded superpositions of oriented layers can be detected and estimated in multispectral images. We expect our results to be useful in applications that involve the processing of multispectral images, e.g. for feature extraction, compression, and denoising. Moreover, our methods show how the detection and estimation of features like orientations, corners, crossing etc. can be improved by the use of multispectral images.

1 Introduction

Let a gray-scale image be modelled by a function $f : \mathbb{R}^2 \to \mathbb{R}$. Given an (open) region Ω, for all $(x, y) \in \Omega$, either (a) $f(x, y) = \text{constant}$; or (b) $f(x, y) = g(ax + by)$, for some g, a, b; or (c) f varies along all directions. The image f is said to locally have intrinsic dimension $0, 1$ or 2, respectively (0D, 1D, 2D for short) [1]. The intrinsic dimension is relevant to image coding due to the predominance of 0D and 1D regions in natural images [2] and the fact that images are fully determined by the 2D regions, i.e. the whole image information is contained in the 2D regions [3]. The concept can be expressed in a more mathematical form as follows [4]. For a given region Ω, we choose a linear subspace $E \subset \mathbb{R}^2$, of highest dimension, such that

$$f(\mathbf{x} + \mathbf{v}) = f(\mathbf{x}) \quad \text{for all } \mathbf{x}, \mathbf{v} \text{ such that } \mathbf{x}, \mathbf{x} + \mathbf{v} \in \Omega,\ \mathbf{v} \in E. \tag{1}$$

The intrinsic dimension of f is therefore $2 - \dim(E)$ for images (and $n - \dim(E)$ for n-dimensional signals). The intrinsic dimension can be estimated with differential methods, and we will review three such methods below. More general approaches are based on the compensation principle [1] and the Volterra-Wiener theory of nonlinear systems [5].

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Structure Tensor. This is a straightforward method based on the equivalence in Ω of Eq. (1) and the constraint

$$\frac{\partial f}{\partial \mathbf{v}} = 0 \quad \text{for all } \mathbf{v} \in E \ . \tag{2}$$

The subspace E can be estimated as the subspace spanned by the set of unity vectors that minimise the energy functional

$$\mathcal{E}(\mathbf{v}) = \int_\Omega \left| \frac{\partial f}{\partial \mathbf{v}} \right|^2 \mathrm{d}\Omega = \mathbf{v}^T \mathbf{J} \mathbf{v} \ , \tag{3}$$

where $\mathbf{J}$ is given by

$$\mathbf{J} = \int_\Omega \nabla f \otimes \nabla f \ \mathrm{d}\Omega = \int_\Omega \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix} \mathrm{d}\Omega \ . \tag{4}$$

In the above equation, the symbol $\otimes$ denotes the tensor product, and $f_x,\ f_y$ are short notations for $\partial f/\partial x,\ \partial f/\partial y$. Therefore, E is the eigenspace associated with the smallest eigenvalues of $\mathbf{J}$, and the intrinsic dimension of f corresponds to the rank of $\mathbf{J}$ and may be obtained from the eigenvalue analysis of $\mathbf{J}$ or, equivalently, from its symmetric invariants [6].

The Hessian. Since Eq. (1) is assumed to be valid in a neighbourhood, it follows that, in Ω,

$$\frac{\partial^2 f}{\partial \mathbf{w} \partial \mathbf{v}} = \mathbf{0} \quad \text{for all } \mathbf{v} \in E \text{ and } \mathbf{w} \in \mathbb{R}^2 \tag{5}$$

or, equivalently,

$$\mathbf{H}\mathbf{v} = \mathbf{0} \quad \text{for all } \mathbf{v} \in E \ , \ [7] \tag{6}$$

where $\mathbf{H}$ is the Hessian of f, i.e.,

$$\mathbf{H} = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix}. \tag{7}$$

Hence, as for the structure tensor method, both the subspace E and the intrinsic dimension can be estimated from the eigenvalue analysis of the Hessian of f [1].

The Energy Tensor. The structure-tensor and Hessian methods have similar drawbacks. The first fails at singular points, e.g., extreme points, while the second fails at inflection points of the image. Equations (2) and (5) may be combined into a phase invariant tensor, the so called *energy tensor* [8]:

$$\mathbf{B} = \nabla f \otimes \nabla f - f\mathbf{H}. \tag{8}$$

Note that the energy tensor is a combination of the structure tensor and the Hessian.

The purpose of this paper is twofold: first, we show how the above methods for the estimation of the intrinsic dimension generalise to multispectral images; second, we extend the methods to a different class of signals with fractional intrinsic dimension, which occur, for example, with multiple overlayed orientations and occlusions, see Figure 1.

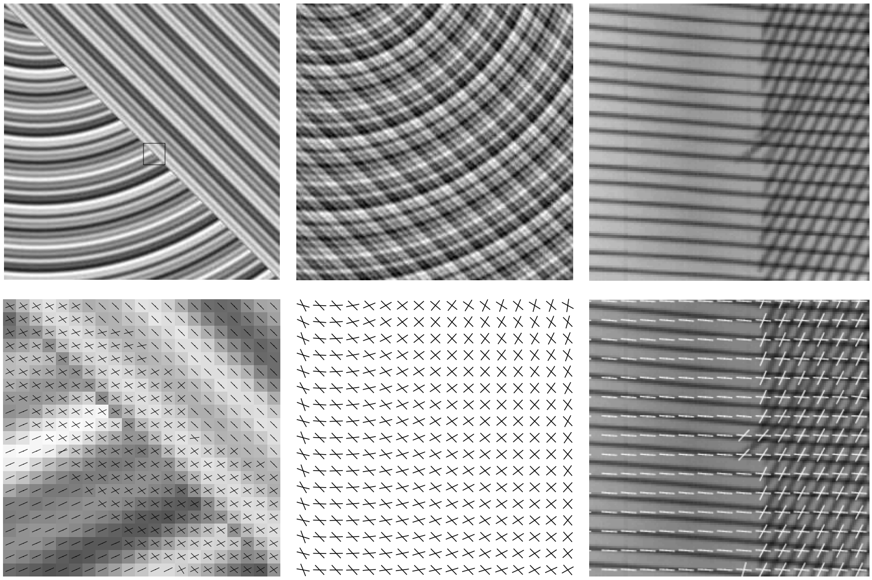

Fig. 1. Examples of images that result from combinations of locally 1D layers. Synthetic examples of occlusion (top left) and transparent overlay (top middle), and a real example of an X-ray car-tire image (top right) are shown. In the bottom row the results of estimating the orientations are shown. Results are for grey-level images and are taken from [9]. Note that in the case of multispectral images, the results will improve because the structure tensor can be built by integration in colour space instead of image space.

2 Multispectral Images and Intrinsic Dimension

Let a multispectral image be modelled by a function $\mathbf{f} : \mathbb{R}^2 \rightarrow \mathbb{R}^q$. The concept of intrinsic dimension extends straightforwardly to such images: a 0D image has constant colour; in 1D images, the colours are constant along some direction; otherwise, the image is 2D. Note that now the colours, not the grey levels, of the image must be constant in a subspace in order to have a reduced intrinsic dimension. Next, we show how to extend the three differential methods described above to the case of multispectral images. Since the coordinate system $\mathbb{R}^q$ is mainly an artifact of the colour space definition, it seems useful to have a generalisation that does not depend on a particular choice of such a coordinate system. In particular, just working with the image components independently does not seem appropriate because they depend on the chosen colour space.

Structure Tensor. Similar to the case of scalar images, we look for the subspace E of highest dimension such that, in Ω,

$$\frac{\partial \mathbf{f}}{\partial \mathbf{v}} = \mathbf{0} \quad \text{for all } \mathbf{v} \in E\,. \tag{9}$$

This leads to a system of q equations for each $\mathbf{x} \in \Omega$. Since Eq. (9) is a vectorial equation, we choose a scalar product (and its corresponding norm) in $\mathbb{R}^q$ for $\mathbf{y} =$

$(y_1, \ldots, y_q)$ and $\mathbf{z} = (z_1, \ldots, z_q)$ in $\mathbb{R}^q$ as $\mathbf{y} \cdot \mathbf{z} = \sum_1^q a_k y_k z_k$. The positive weights a_k can emphasise some components of $\mathbf{f}$ relative to others. We may now estimate E and the intrinsic dimension of $\mathbf{f}$ by minimising the functional

$$\mathcal{E}(\mathbf{v}) = \int_\Omega \left\| \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \right\|^2 \mathrm{d}\Omega \, . \tag{10}$$

To find the tensor $\mathbf{J}$ associated with $\mathcal{E}$, we set $\mathbf{v} = (v_x, v_y)$. Thus,

$$\frac{\partial \mathbf{f}}{\partial \mathbf{v}} = v_x \mathbf{f}_x + v_y \mathbf{f}_y, \tag{11}$$

where $\mathbf{f}_x, \mathbf{f}_y$ denote the partial derivatives of $\mathbf{f}$. Therefore,

$$\left\| \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \right\|^2 = v_x^2 \|\mathbf{f}_x\|^2 + 2 v_x v_y \mathbf{f}_x \cdot \mathbf{f}_y + v_y^2 \|\mathbf{f}_y\|^2. \tag{12}$$

Hence, Eq. (10) can be rewritten as

$$\mathcal{E}(\mathbf{v}) = \mathbf{v}^T \mathbf{J} \mathbf{v}, \tag{13}$$

where $\mathbf{J} = \mathbf{J}(\mathbf{f})$ is defined as

$$\mathbf{J} = \int_\Omega \begin{bmatrix} \|\mathbf{f}_x\|^2 & \mathbf{f}_x \cdot \mathbf{f}_y \\ \mathbf{f}_x \cdot \mathbf{f}_y & \|\mathbf{f}_y\|^2 \end{bmatrix} \mathrm{d}\Omega. \tag{14}$$

Thus, as in the gray-scale case, E and the intrinsic dimension of $\mathbf{f}$ may be estimated from the eigenvalue analysis of $\mathbf{J}$. Similar results have been obtained for the the gradient of colour images [10] and for motion from colour [7]. As expected, the tensor in Eq. (14) reduces to the one in Eq. (4) for gray-scale images. We relate the structure tensor of $\mathbf{f}$ and its components by the following

Proposition 1. *Let* $\mathbf{f} = (f_1, \ldots, f_q)^T$, *then* $\mathbf{J}(\mathbf{f}) = \sum_1^q a_k \mathbf{J}(f_k)$.

Proof. We have $\partial \mathbf{f}(\mathbf{x}) / \partial \mathbf{v} = (\nabla f_1 \cdot \mathbf{v}, \ldots, \nabla f_q \cdot \mathbf{v})^T$, and consequently,

$$\left\| \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{v}} \right\|^2 = \sum_1^q a_k |\nabla f_k \cdot \mathbf{v}|^2 = \mathbf{v}^T \big(\sum_1^q a_k \nabla f_k \otimes \nabla f_k \big) \mathbf{v} \, , \tag{15}$$

which yields the result by use of the scalar product defined above.

The Hessian. We start by taking directional derivatives of Eq. (9) to obtain, within Ω,

$$\frac{\partial^2 \mathbf{f}}{\partial \mathbf{w} \partial \mathbf{v}} = \mathbf{0} \quad \text{for all } \mathbf{v} \in E \text{ and } \mathbf{w} \in \mathbb{R}^2 \, . \tag{16}$$

Let $\mathbf{H}_k = \mathbf{H}(f_k)$ be the Hessian of the component f_k , then we have

$$\frac{\partial^2 \mathbf{f}}{\partial \mathbf{w} \partial \mathbf{v}} = (\mathbf{w}^T \mathbf{H}_1 \mathbf{v}, \ldots, \mathbf{w}^T \mathbf{H}_q \mathbf{v})^T \, . \tag{17}$$

By an abuse of notation, we denote by the Hessian of $\mathbf{f}$ the linear mapping $\mathbf{H} : \mathbb{R}^2 \to \mathbb{R}^{2q} = \mathbb{R}^q \times \cdots \times \mathbb{R}^q$ defined by

$$\mathbf{H}\mathbf{u} = (\mathbf{H}_1\mathbf{u}, \ldots, \mathbf{H}_q\mathbf{u})^T . \tag{18}$$

With this notation, Eq. (16) holds if and only if

$$\mathbf{H}\mathbf{v} = \mathbf{0} \quad \text{for all } \mathbf{v} \in E . \tag{19}$$

Since the above equation is overconstrained for $q > 1$, it has to be solved in a least-squares sense, which yields E as the subspace spanned by the unity eigenvectors associated to the minimal eigenvalue of $\mathbf{H}^T\mathbf{H}$. Thus, the intrinsic dimension of $\mathbf{f}$ may be obtained as the rank of $\mathbf{H}^T\mathbf{H}$. By choosing, in $\mathbb{R}^2$, the standard scalar product and, for $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_q)$, $\mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_q)$ in $\mathbb{R}^{2q}$, the scalar product $\mathbf{y} \cdot \mathbf{z} = \sum_1^q a_k \, \mathbf{y}_k \cdot \mathbf{z}_k$, a straightforward computation gives

Proposition 2. $\mathbf{H}^T\mathbf{H} = \sum_1^q a_k \mathbf{H}^2(f_k)$.

The Energy Tensor. The energy tensor method for the estimation of intrinsic dimension can be extended straightforwardly to multispectral images in analogy to the Hessian case. We define a linear mapping $\mathbf{B} : \mathbb{R}^2 \to \mathbb{R}^{2q}$ by taking into account the energy tensor for the coordinate functions of $\mathbf{f}$, i.e.

$$\mathbf{B}\mathbf{u} = (\mathbf{B}_1\mathbf{u}, \ldots, \mathbf{B}_q\mathbf{u})^T , \tag{20}$$

where $\mathbf{B}_k = \mathbf{B}(f_k) = \nabla f_k \otimes \nabla f_k - f_k \mathbf{H}(f_k)$. Therefore, the subspace E is the kernel of $\mathbf{B}$, i.e.

$$\mathbf{B}\mathbf{v} = \mathbf{0} \text{ for all } \mathbf{v} \in E . \tag{21}$$

Thus, again, E is estimated as the eigenspace associated to the minimal eigenvalue of $\mathbf{B}^T\mathbf{B}$, and the intrinsic dimension of $\mathbf{f}$ is computed as the rank of $\mathbf{B}^T\mathbf{B}$. As before, we have the following

Proposition 3. $\mathbf{B}^T\mathbf{B} = \sum_1^q a_k \mathbf{B}^2(f_k)$.

3 Intrinsic Dimension of Multiple, Multispectral Layers

In this section, we investigate a special class of 2D images which occur when two 1D layers are combined into one 2D image, e.g. by additive superposition or occlusion. We will show that a generalised structure tensor can be used to detect such superpositions and to estimate the parameters of the 1D layers. Because such combinations of 1D layers have an intrinsic dimension greater than one, but are not really 2D, we say that they have a *fractional intrinsic dimension* between 1 and 2 and, thus, need a more refined description (in terms of the generalised structure tensor introduced below).

Pattern	rank $\mathbf{J}$	rank $\mathbf{J}_2$
○	0	0
|	1	1
|+|	2	2
others	2	3

Table 1. The shown correspondences between the different patterns and the ranks of the two tensors define the intrinsic dimension of the components of additively overlayed images. The symbols denote $0D$ (circle) and $1D$ (bar) patterns. In general, the rank of $\boldsymbol{J}_N$, $N = 1, 2, ...$ induces a natural order of complexity for patterns consisting of N additive layers [11].

Additive Multispectral Layers. Let $\mathbf{f}$ be the additive superposition of two layers

$$\mathbf{f} = \mathbf{g} + \mathbf{h}\,, \tag{22}$$

we want to know if both layers $\mathbf{g}, \mathbf{h}$ have intrinsic dimension lower than two, i.e., for a given region Ω, we want to know if there are subspaces E_1 and E_2 such that

$$\frac{\partial \mathbf{g}}{\partial \mathbf{u}} = \frac{\partial \mathbf{h}}{\partial \mathbf{v}} = \mathbf{0} \quad \text{for all } \mathbf{u} \in E_1,\ \mathbf{v} \in E_2\,. \tag{23}$$

By allowing more layers, we can deal with the important case where the components f_k of $\mathbf{f}$ have intrinsic dimension lower than 2. For this, it suffices to look at $\mathbf{f} = \sum_1^q f_k \mathbf{e}_k$, where $\{\mathbf{e}_1, \ldots, \mathbf{e}_k\}$ is the standard basis for $\mathbb{R}^q$. Taken together, Eq.s (22) and (23) are equivalent to [12]

$$\frac{\partial^2 \mathbf{f}}{\partial \mathbf{u} \partial \mathbf{v}} = \mathbf{u}^T \mathbf{H} \mathbf{v} = \mathbf{0}\,, \tag{24}$$

which expands to

$$c_{xx}\mathbf{f}_{xx} + c_{xy}\mathbf{f}_{xy} + c_{yy}\mathbf{f}_{yy} = \mathbf{0}, \tag{25}$$

where $c_{xx} = u_x v_x\,, c_{xy} = u_x v_y + u_y v_x\,, c_{yy} = u_y v_y$. Since the above equation is linear in the parameter vector $\mathbf{c} = (c_{xx}, c_{xy}, c_{yy})^T$, there will be a correspondence between the dimension of the subspaces E_1, E_2 and the rank of the tensor associated to the energy functional (see [11] and Table 1)

$$\mathcal{E}_2(\mathbf{c}) = \int_\Omega \|c_{xx}\mathbf{f}_{xx} + c_{xy}\mathbf{f}_{xy} + c_{yy}\mathbf{f}_{yy}\|^2 \,\mathrm{d}\Omega = \mathbf{c}^T \mathbf{J}_2 \mathbf{c}, \tag{26}$$

where $\mathbf{J}_2 = \mathbf{J}_2(\mathbf{f})$ is given by

$$\mathbf{J}_2 = \int_\Omega \begin{bmatrix} \|\mathbf{f}_{xx}\|^2 & \mathbf{f}_{xx}\cdot\mathbf{f}_{xy} & \mathbf{f}_{xx}\cdot\mathbf{f}_{yy} \\ \mathbf{f}_{xx}\cdot\mathbf{f}_{xy} & \|\mathbf{f}_{xy}\|^2 & \mathbf{f}_{xy}\cdot\mathbf{f}_{yy} \\ \mathbf{f}_{xx}\cdot\mathbf{f}_{yy} & \mathbf{f}_{xy}\cdot\mathbf{f}_{yy} & \|\mathbf{f}_{yy}\|^2 \end{bmatrix} \mathrm{d}\Omega\,. \tag{27}$$

Multiplicative Multispectral Layers. We now consider the multiplicative superposition of two layers, i.e.

$$\mathbf{f} = \mathbf{g} \bullet \mathbf{h}, \tag{28}$$

where the bullet denotes that for every component of $\mathbf{f}$, we have $f_k = g_k h_k$. A direct verification shows that Eq.s (23) and (28) imply the following constraint for $\mathbf{f}$:

$$\mathbf{u}^T \mathbf{B} \mathbf{v} = \mathbf{0}. \tag{29}$$

In analogy to the additive case, we can construct a tensor $\mathbf{J}(\mathbf{B}, \mathbf{f})$ for the estimation of the intrinsic dimension of the layers.

Occluded Multispectral Layers. We model occluded superposition of two images by

$$\mathbf{f} = \chi \mathbf{g} + (1 - \chi)\mathbf{h} \,, \tag{30}$$

where $\chi(\mathbf{x})$ is the characteristic function of some half-plane P. This model is appropriate for the local description of junction types T, L and Ψ. X-junctions fit better to a transparent model as in Subsection 3. In order to estimate the intrinsic dimension of the occlusion layers, we observe that Eq. (30) is equivalent to

$$\mathbf{f}(\mathbf{x}) = \begin{cases} \mathbf{g}_1(\mathbf{x}) \text{ if } \mathbf{x} \in P \\ \mathbf{g}_2(\mathbf{x}) \text{ otherwise.} \end{cases} \tag{31}$$

Therefore, $\partial \mathbf{f}(\mathbf{x})/\partial \mathbf{u} = \mathbf{0}$ if $\mathbf{x}$ belongs to P , and $\partial \mathbf{f}(\mathbf{x})/\partial \mathbf{v} = \mathbf{0}$ if $\mathbf{x}$ does not belong to P. From the above, we can draw the conclusion that the expressions

$$\frac{\partial \mathbf{f}}{\partial \mathbf{u}} \otimes \frac{\partial \mathbf{f}}{\partial \mathbf{v}} = \mathbf{0} \quad \text{and} \quad \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \otimes \frac{\partial \mathbf{f}}{\partial \mathbf{u}} = \mathbf{0} \tag{32}$$

are valid everywhere except for the border of P where they may differ from zero. Eq. (32) may not hold at the border of P, because the derivatives of the characteristic function χ are not defined there. This is not the case if $\mathbf{u}$ and the border of P have the same direction, e.g., in case of a T-junction. In contrast to the second order derivative approach, the equations in (32) differ. Expanding the first of these equations, we find

$$u_x v_x \mathbf{f}_x \otimes \mathbf{f}_x + u_x v_y \mathbf{f}_x \otimes \mathbf{f}_y + u_y v_x \mathbf{f}_y \otimes \mathbf{f}_x + u_y v_y \mathbf{f}_y \otimes \mathbf{f}_y = \mathbf{0} \,. \tag{33}$$

A direct estimation of the $u_i v_j$ will result in an overconstrained system of equations for $\mathbf{u}$ and $\mathbf{v}$. This is avoided by averaging the equations in (32), i.e., by symmetrization, to obtain

$$c_{xx} \mathbf{f}_x \otimes \mathbf{f}_x + \frac{c_{xy}}{2} (\mathbf{f}_x \otimes \mathbf{f}_y + \mathbf{f}_y \otimes \mathbf{f}_x) + c_{yy} \mathbf{f}_y \otimes \mathbf{f}_y = \mathbf{0} \,. \tag{34}$$

Although the focus here is not on the estimation of the parameters $\mathbf{u}$ and $\mathbf{v}$, the symmetrization has the extra benefit of reducing the size of the resulting tensor. The system in Eq. (34) has $q(q+1)/2$ equations, which makes the system overconstrained if $q > 2$. Note that the system is underconstrained for $q = 1$ and that a multispectral approach can overcome this problem. As in the case of transparent layers, a least-squares procedure to solve Eq. (34) will lead to the minima of the energy functional

$$\begin{aligned} \mathcal{E}(\mathbf{c}) &= \int_\Omega \left\| c_{xx} \mathbf{f}_x \otimes \mathbf{f}_x + \frac{c_{xy}}{2} (\mathbf{f}_x \otimes \mathbf{f}_y + \mathbf{f}_y \otimes \mathbf{f}_x) + c_{yy} \mathbf{f}_y \otimes \mathbf{f}_y \right\|^2 \mathrm{d}\Omega & (35) \\ &= \mathbf{c} \mathbf{J}_2 \mathbf{c} \,, & (36) \end{aligned}$$

where

$$\mathbf{J}_2 = \int_\Omega \begin{bmatrix} \|\mathbf{f}_x\|^4 & \|\mathbf{f}_x\|^2 \mathbf{f}_x \cdot \mathbf{f}_y & |\mathbf{f}_x \cdot \mathbf{f}_y|^2 \\ \|\mathbf{f}_x\|^2 \mathbf{f}_x \cdot \mathbf{f}_y & \frac{1}{2}\left(\|\mathbf{f}_x\|^2 \|\mathbf{f}_y\|^2 + |\mathbf{f}_x \cdot \mathbf{f}_y|^2\right) & \|\mathbf{f}_y\|^2 \mathbf{f}_x \cdot \mathbf{f}_y \\ |\mathbf{f}_x \cdot \mathbf{f}_y|^2 & \|\mathbf{f}_y\|^2 \mathbf{f}_x \cdot \mathbf{f}_y & \|\mathbf{f}_y\|^4 \end{bmatrix} \mathrm{d}\Omega \,. \tag{37}$$

As before, a correspondence between the intrinsic dimension of the occlusion layers and the rank of $\mathbf{J}_2$ is given by Table 1.

4 Discussion

We have addressed the basic question of how to encode local signal variation in the case of multispectral images. The results remain valid for any vector-valued two-dimensional signal and can be extended to n-dimensional signals [9]. We have shown how the concept of intrinsic dimension and the estimation of multiple orientations can be applied to multispectral images in a way which does not depend on the chosen colour space. We expect this to be useful for those who work with multispectral images and need to extract meaningful features, compress, or denoise such images. Moreover, we expect that some may choose to use multispectral images to improve the results obtained with scalar images. This is because our results show how multiple spectral components, if they differ, can help to estimate significant image features like orientations, corners, and junctions. The estimation of local structure always requires a certain neighbourhood and multispectral images offer the possibility to trade spatial against spectral neighbourhoods. Our methods are based on derivatives, but we have discussed elsewhere, e.g. in [6, 4], that this is not a serious practical restriction. Nevertheless, a more general non-differential theory remains desirable.

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