Learning Orthogonal Bases for k-Sparse Representations

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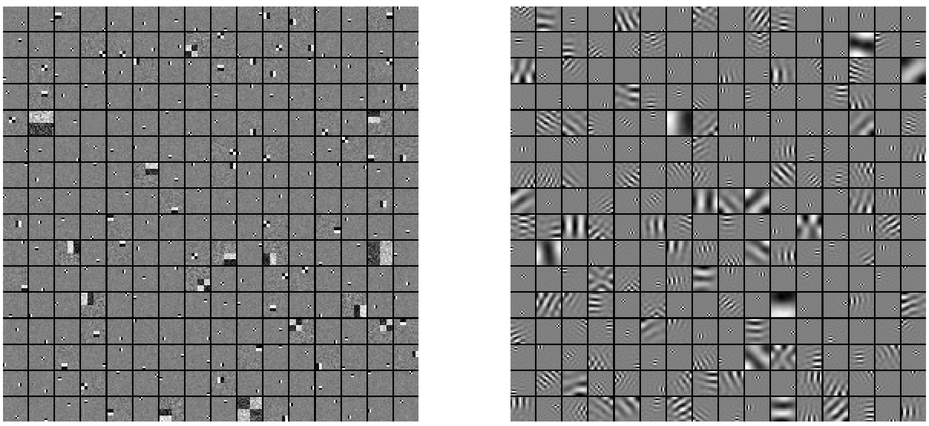
Sparse Coding aims at finding a dictionary for a given data set, such that each sample can be represented by a linear combination of only few dictionary atoms. Generally, sparse coding dictionaries are overcomplete and not orthogonal. Thus, the processing substep to determine the optimal k-sparse representation of a given sample by the current dictionary is NP-hard. Usually, the solution is approximated by a greedy algorithm or by l_1 convex relaxation. With an orthogonal dictionary, however, an optimal k-sparse representation can not only be efficiently, but exactly computed, because a corresponding k-sparse coefficient vector is given by the k largest absolute projections.

In this paper, we present the novel online learning algorithm Orthogonal Sparse Coding (OSC), that is designed to find an orthogonal basis $U = (\boldsymbol{u}_1, ..., \boldsymbol{u}_d)$ for a given data set $X \in \mathbb{R}^{d \times L}$, such that for any $k \in \{1, ..., d\}$, the optimal k-sparse coefficient vectors $A \in \mathbb{R}^{d \times L}$ minimize the average representation error $E = \frac{1}{dL} \|X - UA\|_\mathrm{F}^2$. At each learning step t, OSC randomly selects a sample $\boldsymbol{x}$ from X and determines an index sequence $h_1, ..., h_d$ of decreasing overlaps $|\boldsymbol{u}_{h_i}^\mathrm{T} \boldsymbol{x}|$ between $\boldsymbol{x}$ and the basis vectors in U. In the order of that sequence, each



(a) Learned basis from 1,000 *synthetic* image patches of size 16×16 pixel.

(b) Learned basis from 20,000 *natural* image patches of size 16×16 pixel.

Fig. 1: Basis patches learned with OSC.

basis vector $\boldsymbol{u}_{h_i}$ is updated by the Hebbian learning rule $\Delta \boldsymbol{u}_{h_i} = \varepsilon_t(\boldsymbol{u}_{h_i}^{\mathrm{T}} \boldsymbol{x})\boldsymbol{x}$ with a subsequent unit length normalization. After each basis vector update, $\boldsymbol{x}$ and the next basis vector $\boldsymbol{u}_{h_{i+1}}$ to be adapted are projected onto the orthogonal complement $\mathrm{span}(\{\boldsymbol{u}_{h_1}, ..., \boldsymbol{u}_{h_i}\})^{\perp}$ wherein the next update takes place.

We applied OSC to (*i*) 1,000 synthetic (k=50)-sparse patches of size 16×16 pixel, randomly generated with a 2D Haar basis, and (*ii*) 20,000 natural image patches of size 16×16 pixel, that were randomly sampled from the first image set of the nature scene collection [1] (308 images of nature scenes containing no man-made objects or people). The basis patches learned by OSC are shown in Figure 1 and demonstrate that OSC reliably recovers the generating basis from synthetic data (see Figure 1a). Figure 1b illustrates that the OSC basis learned on the natural image patches resembles a wavelet decomposition, and is distinct from PCA, DCT, and Haar bases.

In Figure 2, the average k-term approximation performance of the OSC basis is compared with PCA, DCT, Haar and JPEG 2000 wavelets on the natural image patch data set. For this data set, OSC yields a consistently better k-term approximation performance than any of the alternative methods.

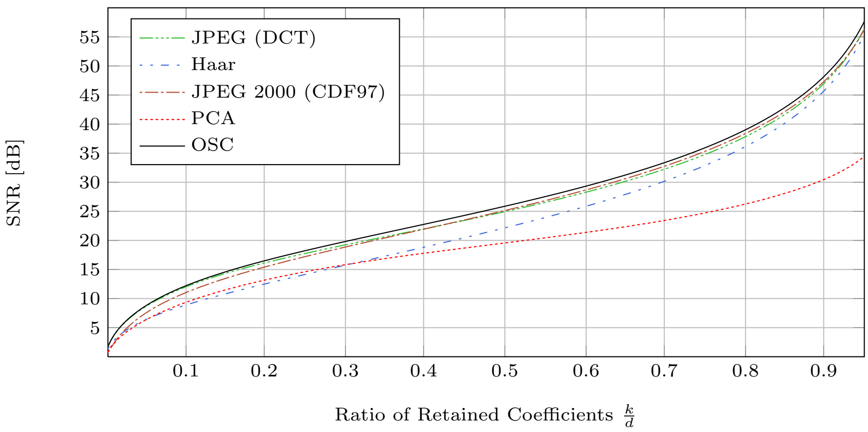


Fig. 2: Average k-term approximation performance of 20,000 natural image patches of size 16×16 pixel.

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References

1. Wilson S. Geisler and Jeffrey S. Perry. Statistics for optimal point prediction in natural images. *Journal of Vision*, 11(12), October 2011.